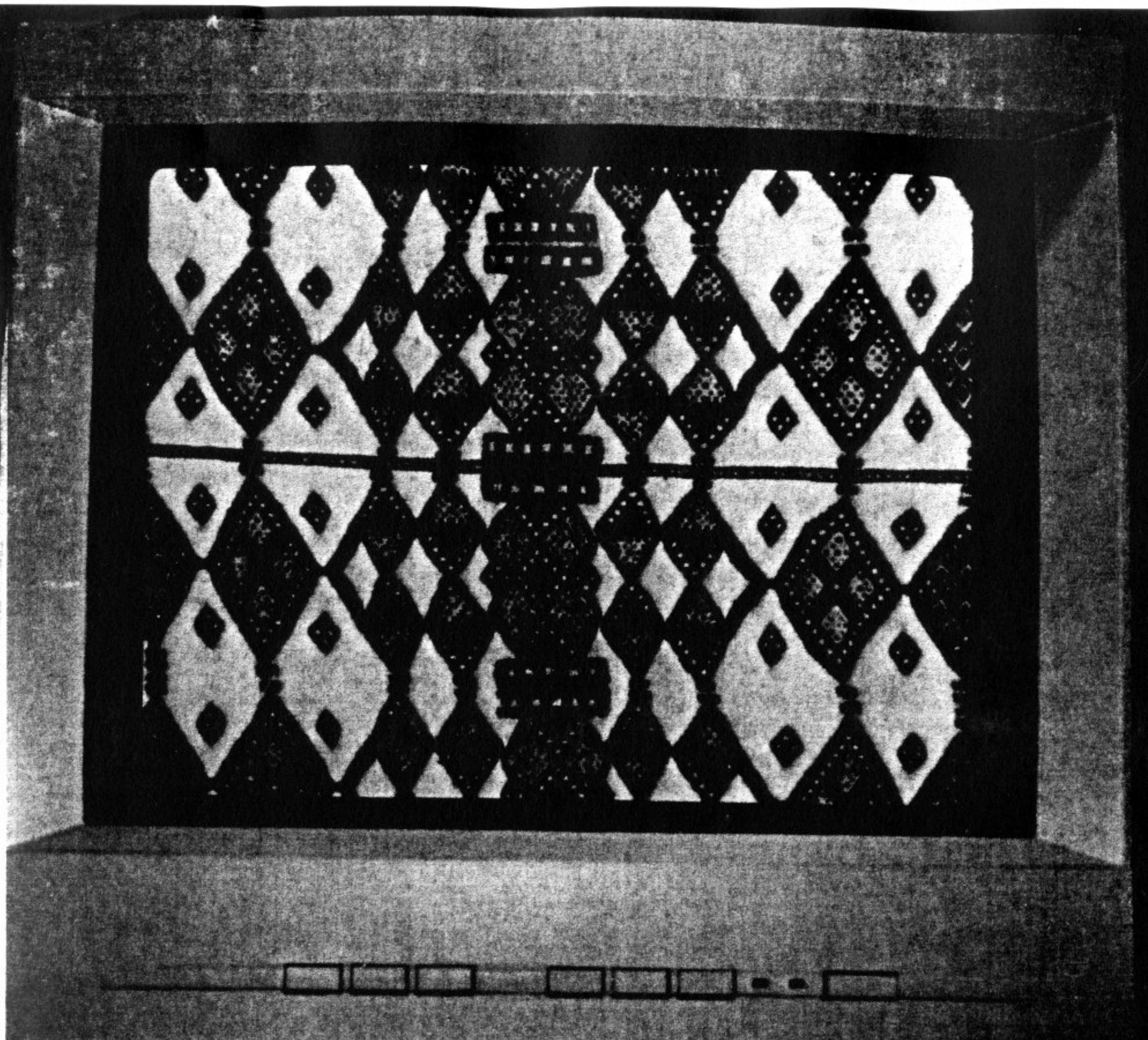


African Fractals

MODERN COMPUTING AND INDIGENOUS DESIGN

RON EGLASH

CHAPTER I

Introduction to fractal geometry

Fractal geometry has emerged as one of the most exciting frontiers in the fusion between mathematics and information technology. Fractals can be seen in many of the swirling patterns produced by computer graphics, and they have become an important new tool for modeling in biology, geology, and other natural sciences. While fractal geometry can indeed take us into the far reaches of high-tech science, its patterns are surprisingly common in traditional African designs, and some of its basic concepts are fundamental to African knowledge systems. This book will provide an easy introduction to fractal geometry for people without any mathematics background, and it will show how these same categories of geometric pattern, calculation, and theory are expressed in African cultures.

Mathematics and culture

For many years anthropologists have observed that the patterns produced in different cultures can be characterized by specific design themes. In Europe and America, for example, we often see cities laid out in a grid pattern of straight streets and right-angle corners. Another grid, the Cartesian coordinate system, has long been a foundation for the mathematics used in these societies. In many works

of Chinese art we find hexagons used with extraordinary geometric precision—a choice that might seem arbitrary were it not for the importance of the number six in the hexagrams of their fortunetelling system (the *I Ching*), in the anatomy charts for acupuncture (*liù-qi* or "six spirits"), and even in Chinese architecture.[1] Shape and number are not only the universal rules of measurement and logic; they are also cultural tools that can be used for expressing particular social ideas and linking different areas of life. They are, as Claude Lévi-Strauss would put it, "good to think with."

Design themes are like threads running through the social fabric; they are less a commanding force than something we command, weaving these strands into many different patterns of meaning. The ancient Chinese empires, for example, used a base-10 counting system, and they even began the first universal metric system.[2] So the frequent use of the number 60 in Chinese knowledge systems can be linked to the combination of this official base 10 notation with their sacred number six. In some American cities we find that the streets are numbered like Cartesian coordinates, but in others they are named after historical figures, and still others combine the two. These city differences typically correspond to different social meanings—an emphasis on history versus efficiency, for example.

Suppose that visitors from another world were to view the grid of an American city. For a city with numbered streets, the visitors (assuming they could read our numbers) could safely conclude that Americans made use of a coordinate structure. But do these Americans actually understand coordinate mathematics? Can they use a coordinate grid to graph equations? Just how sophisticated is their mathematical understanding? In the following chapter, we will find ourselves in a similar position, for African settlement architecture is filled with remarkable examples of fractal structure. Did precolonial Africans actually understand and apply fractal geometry?

As I will explain in this chapter, fractals are characterized by the repetition of similar patterns at ever-diminishing scales. Traditional African settlements typically show this "self-similar" characteristic: circles of circles of circular dwellings, rectangular walls enclosing ever-smaller rectangles, and streets in which broad avenues branch down to tiny footpaths with striking geometric repetition. The fractal structure will be easily identified when we compare aerial views of these African villages and cities with corresponding fractal graphics simulations.

What are we to make of this comparison? Let's put ourselves back in the shoes of the visitors from another planet. Having beamed down to an American settlement named "Corvallis, Oregon," they discover that the streets are not numbered, but rather titled with what appear to be arbitrary names: Washington, Jefferson, Adams, and so on. At first they might conclude that there is nothing mathematical about it. By understanding a bit more about the cultural meaning, however, a mathematical pattern does emerge: these are names in historical succession. It might be only ordering in terms of position in a series (an "ordinal" number), but there is some kind of coordinate system at work after all. African designs have to be approached in the same way. We cannot just assume that African fractals show an understanding of fractal geometry, nor can we dismiss that possibility. We need to listen to what the designers and users of these structures have to say about it. What appears to be an unconscious or accidental pattern might actually have an intentional mathematical component.

Overall, the presence of mathematics in culture can be thought of in terms of a spectrum from unintentional to self-conscious. At one extreme is the emergence of completely unconscious structures. Termite mounds, for example, are excellent fractals (they have chambers within chambers within chambers) but no one would claim that termites understand mathematics. In the same way, patterns appear in the group dynamics of large human populations, but these are generally not patterns of which any individual is aware. Unconscious structures do not count as mathematical knowledge, even though we can use mathematics to describe them.

Moving along this spectrum toward the more intentional, we next find examples of decorative designs which, although consciously created, have no explicit knowledge attached to them. It is possible, for example, that an artist who does not know what the word "hexagon" means could still draw one with great precision. This would be a conscious design, but the knowledge is strictly implicit.[3] In the next step along our spectrum, people make these components explicit—they have names for the patterns they observe in shapes and numbers. Taking the intention spectrum one more step, we have rules for how these patterns can be combined. Here we can find "applied mathematics." Of course there is a world of difference between the applied math of a modern engineer and the applied math of a shopkeeper—whether or not something is intentional tells us nothing about its complexity.

Finally we move to "pure mathematics," as found in the abstract theories of modern academic mathematicians. Pure math can also be very simple—for example, the distinction between ordinal numbers (first, second, third) and cardinal numbers (one, two, three) is an example of pure math. But it would not be enough for people in a society simply to use examples of both types; they would have to have words for these two categories and explicitly reflect on a comparison of their properties before we would say that they have a theory of

the distinction between ordinal and cardinal numbers. While applied mathematics makes use of rules, pure math tells us why they work—and how to find new ones.

This book begins by moving along the spectrum just described. We will start by showing that African fractals are not simply due to unconscious activity. We will then look at examples where they are conscious but implicit designs, followed by examples in which Africans have devised explicit rules for generating these patterns, and finally to examples of abstract theory in these indigenous knowledge systems. The reason for taking such a cautious route can be expressed in terms of what philosopher Karl Popper called "falsifiability." Popper pointed out that everyone has the urge to confirm their favorite theories; and so we have to take precautions not to limit our attention to success—a theory is only good if you try to test it for failure. If we only use examples where African knowledge systems successfully matched fractal geometry, we would not know its limitations. There are indeed gaps where the family of theories and practices centered around fractal geometry in high-tech mathematics has no counterpart in traditional Africa. Although such gaps are significant, they do not invalidate the comparison, but rather provide the necessary qualifications to accurately characterize the indigenous fractal geometry of Africa.

Overview of the text

Following the introduction to fractal geometry in the next section, in chapter 2 we will explore fractals in African settlements. It will become clear that the explanation of unconscious group activity does not fit this case. When we talk to the indigenous architects, they are quite explicit about those same fractal features we observe, and use several of the basic concepts of fractal geometry in discussing their material designs and associated knowledge systems. Termites may make fractal architectures, but they do not paint abstract models of the structure on its walls or create symbols for its geometric properties. While these introductory examples won't settle all the questions, we will at least have established that these architectural designs should be explained by something more than unintentional social dynamics.

In chapter 3 we will examine another explanation: perhaps fractal settlement patterns are not unique to Africa, and we have simply observed a common characteristic of all non-Western architectures. Here the concept of design themes become important. Anthropologists have found that the design themes found in each culture are fairly distinct—that is, despite the artistic diversity within each society, most of the culture's patterns can be characterized by specific geometric practices. We will see that although fractal designs do occur outside of Africa (Celtic knots, Ukrainian eggs, and Maori rafters have some excellent examples), they are not everywhere. Their strong prevalence in Africa (and in African-influenced southern India) is quite specific.

Chapter 4 returns to this exploration with fractals in African esthetic design. These examples are important for two reasons. First, they remind us that we cannot assume explicit, formal knowledge simply on the basis of a pattern. In contrast to the fractal patterns of African settlement architecture, these aesthetic fractals, according to the artisans, were made "just because it looks pretty that way." They are neither formal systems (no rules to the game) nor do the artisans' report explicit thinking ("I don't know how or why, it just came to me"). Second, they provide one possible route by which a particular set of mathematical concepts came to be spread over an enormous continent. Trade networks could have diffused the fractal aesthetic across Africa, reinforcing a design theme that may have been scattered about in other areas of life. Of course, such origin stories are never certain, and all too easy to invent.

Part II of this book, starting with chapter 5, presents the explicit design methods and symbolic systems that demonstrate fractal geometry as an African knowledge system. As in the introduction to fractals in the first chapter, I will assume the reader has no mathematics background and provide an introduction to any new concepts along with the African versions. We will see that not only in architecture, but in traditional hairstyling, textiles, and sculpture, in painting, carving, and metalwork, in religion, games, and practical craft, in quantitative techniques and symbolic systems, Africans have used the patterns and abstract concepts of fractal geometry.

Chapter 10, the last in part II, is the result of my collaboration with an African physicist, Professor Christian Sina Diatta. A sponsor for the Fulbright fellowship that enabled my fieldwork in west and central Africa, Dr. Diatta took the idea of indigenous fractals and ran with it, moving us in directions that I would never have considered on my own, and still have yet to explore fully.

In the third and final part of this book we will examine the consequences of African fractal geometry: given that it does exist, what are its social implications? Chapter 11 will briefly review previous studies of African knowledge systems. We will see that although several researchers have proposed ideas related to the fractal concept—Henry Louis Gates's "repetition with revision," Léopold Senghor's "dynamic symmetry," and William Fagg's "exponential morphology" are all good examples—there have been specific obstacles that prevented anthropologists and others from taking up these concepts in terms of African mathematics.

Chapter 12 covers the political consequences of African fractals. On the one hand, we will find there is no evidence that geometric form has any *inherent* social meaning. In settlement design, for example, people report both oppressive and liberatory social experiences with fractal architectures. Fractal versus nonfractal ("Euclidean") geometry does not imply good versus bad. On the other hand, people do invest abstract forms with particular local meanings. To take a controversial example, recent U.S. supreme court decisions declared that voting districts cannot have "bizarre" or "highly irregular" shapes, and several of these fractal contours have been replaced by the straight lines of Euclidean form. If fractal settlement patterns are traditional for people of African descent, and Euclidean settlement patterns for Europeans, is it ethnocentric to insist on only Euclidean voting district lines?

Chapter 13 will examine the cultural history of fractal geometry and its mathematical precursors in Europe. We will see that the gaps are not one-sided: just as Africans were missing certain mathematical ideas in their version of fractal geometry, Europeans were equally affected by their own cultural views and have been slow to adopt some of the mathematical concepts that were long championed by Africans. Indeed, there is striking evidence that some of the sources of mathematical inspiration for European fractals were of African origin. The final chapter will move forward in time, highlighting the contemporary versions of fractal design that have been proposed by African architects in Senegal, Mali, and Zambia, and other illustrations of possible fractal futures.

But to understand all this, we must first visit the fractal past.

A historical introduction to fractal geometry

The work of Georg Cantor (1845–1918), which produced the first fractal, the Cantor set (fig. 1.1), proved to be the beginning of a new outlook on infinity. Infinity had long been considered suspect by mathematicians. How can we claim to be using only exact, explicit rules if we have a symbol that vaguely means "the number you would get if you counted forever"? So many mathematicians, starting with Aristotle, had just banned it outright. Cantor showed that it was possible to keep track of the number of elements in an infinite set, and did so in a deceptively simple fashion. Starting with a single straight line, Cantor erased the middle third, leaving two lines. He then carried out the same operation on those two lines, erasing their middles and leaving four lines. In other words, he used a sort of feedback loop, with the end result of one stage brought back as the starting point for the next. This technique is called "recursion." Cantor showed that if this recursive construction was continued forever, it would create an infinite number of lines, and yet would have zero length.

Not only did Cantor reintroduce infinity as a proper object of mathematical study, but his recursive construction could be used as a model for other "pathological curves," such as that created by Helge von Koch in 1904 (figs. 1.2, 1.3). The mathematical properties of these figures were equally perplexing. Small portions looked just like the whole, and these reflections were repeated down to infinitesimal scales. How could we measure the length of the Koch curve? If

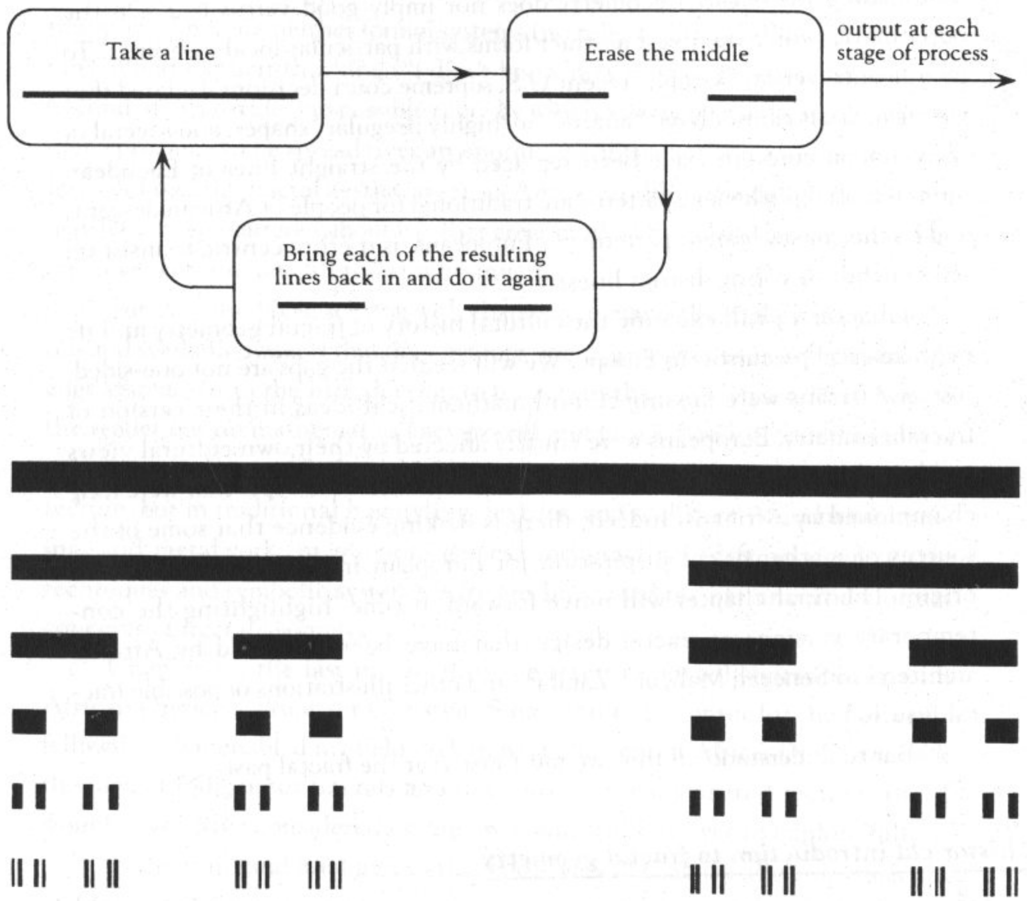


FIGURE 1.1

The Cantor set

In 1877 Georg Cantor came up with the idea of repeatedly subdividing a line to illustrate the concept of an infinite set. This looping technique is called recursion. By specifying that the recursion continues forever, Cantor was able to define an infinite set.

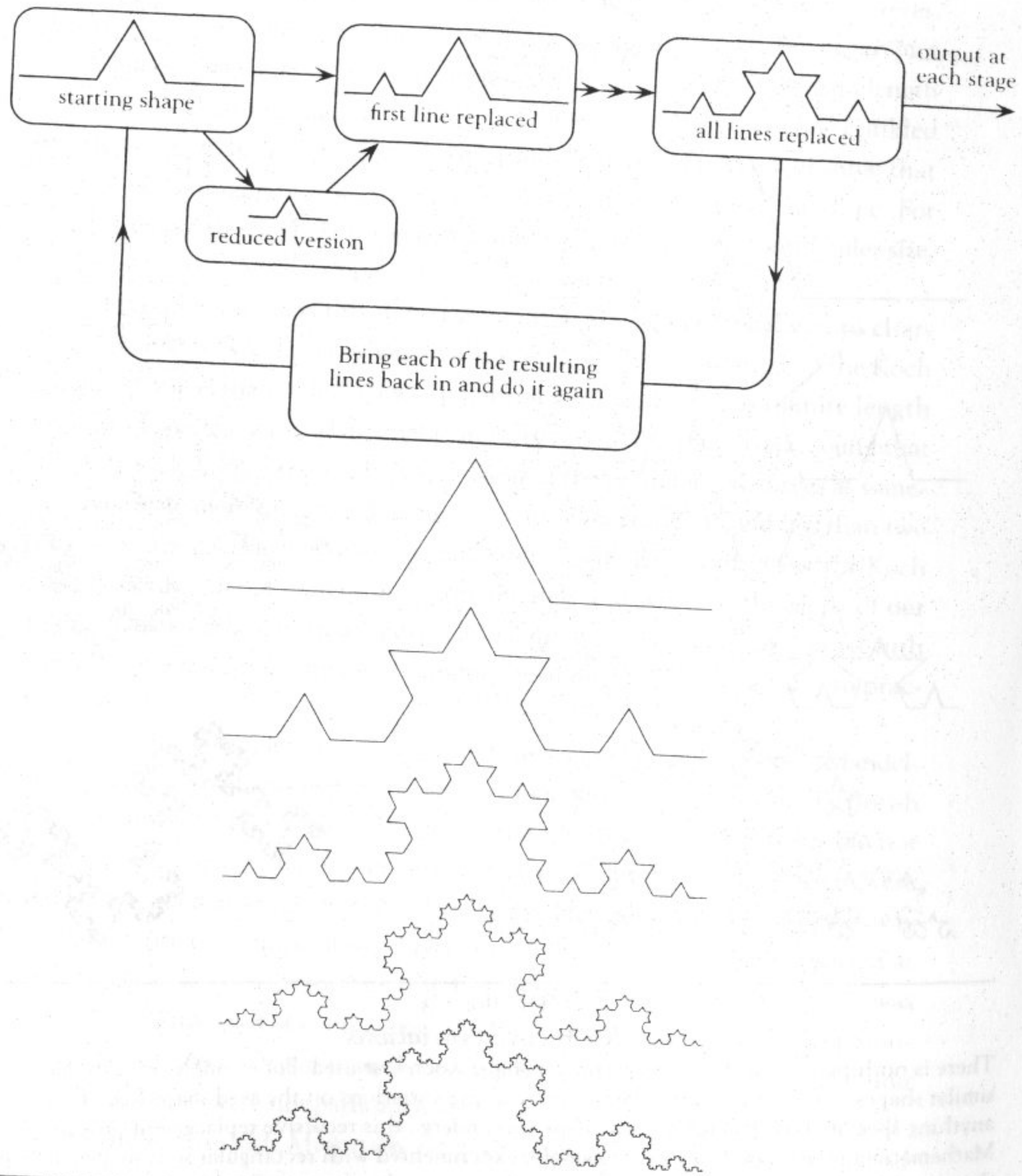


FIGURE 1.2

The Koch curve

Helge von Koch used the same kind of recursive loop as Cantor, but he added lines instead of erasing them. He began with a triangular shape made of four lines, the "seed." He then replaced each of the lines with a reduced version of the original seed shape.

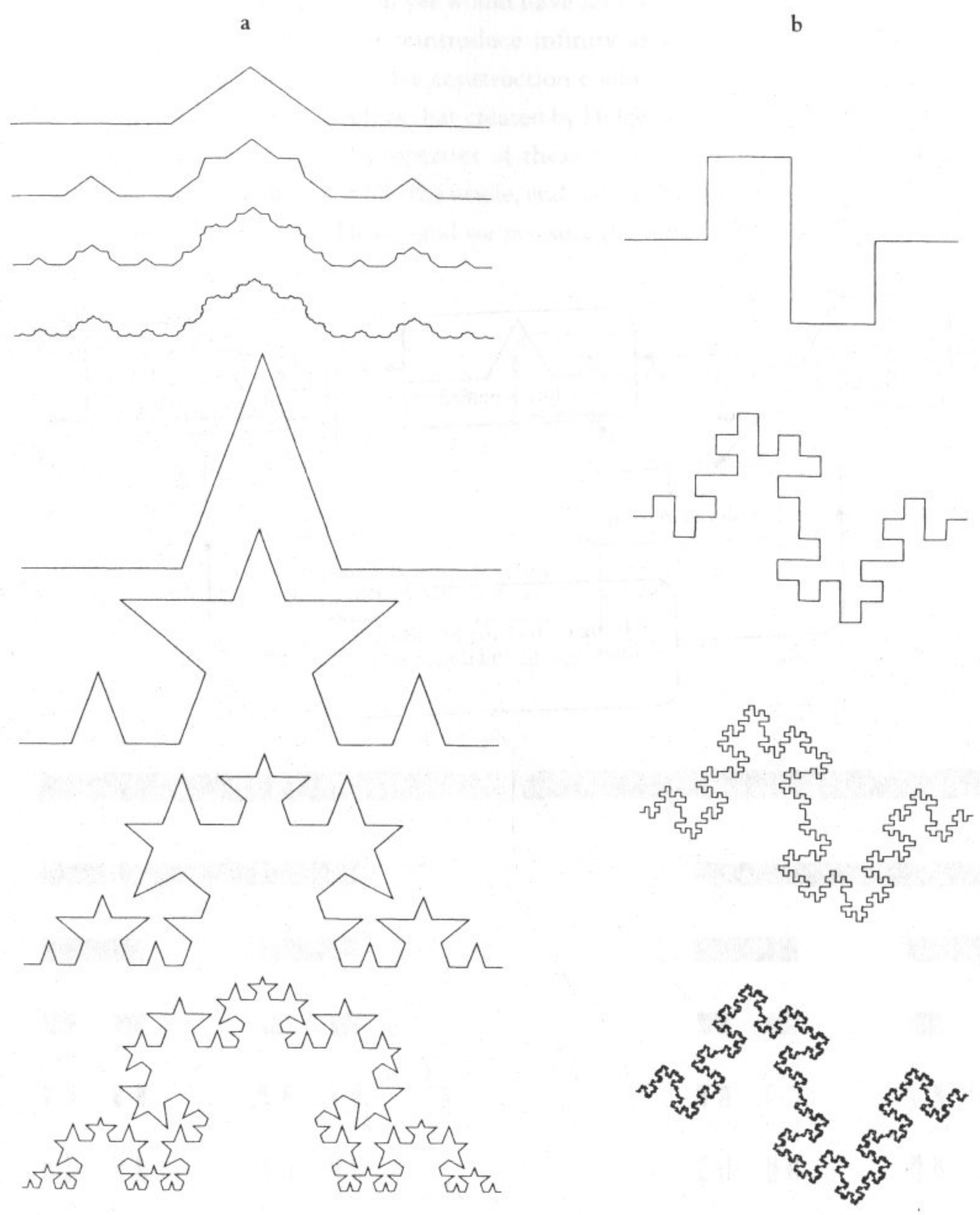


FIGURE 1.3

Koch curve variations

There is nothing special about the particular shape Koch first used. For example, we can make similar shapes that are more flat or more spiked using variations on the seed shape (**a**). Nor is there anything special about triangles—any shape can undergo this recursive replacement process. Mathematician Giuseppe Peano, for example, experimented with rectangular seed shapes such as those in (**b**).

we hold up a six-inch ruler to the curve (fig. 1.4) we get six inches, but of course that misses all the nooks and crannies. If we use a smaller ruler, we get greater length, and with a smaller one even greater length, and so on to infinity. Obviously this is not a very useful way to characterize one of these curves. A new way of thinking about measurement was needed. The answer was to plot these different measures of ruler size versus length, and see how fast we gain length as we shrink the ruler (fig. 1.5). This rate (the slope) tells us just how crinkled or tortuous the curve is. For extremely crinkled curves, the plot will show that we rapidly gain length as we shrink the ruler, so it will have a steep slope. For relatively smooth curves, you don't gain much length as you shrink the ruler size, so the plot has a shallow slope.

To mathematicians this slope was more than just a practical way to characterize crinkles. Recall that when we first tried to measure the length of the Koch curve, we found that its length was potentially infinite. Yet this infinite length fits into a bounded space. Mathematician Felix Hausdorff (1868–1942) found that this paradox could be resolved if we thought of the pathological curves as somehow taking up more than one dimension, as all normal lines do, but less than two dimensions, as flat shapes like squares and circles do. In Hausdorff's view, the Koch curve has a fractional dimension, approximately 1.3, which is the slope of our ruler-versus-length plot. Being pure mathematicians, they were fascinated with this idea of a fractional dimension and never thought about putting it to practical use.

The conceptual leap to practical application was created by Benoit Mandelbrot (b. 1924), who happened upon a study of long-term river fluctuations by British civil servant H. E. Hurst. Hurst had found that the yearly floods of rivers did not have any one average, but rather varied over many different scales—there were flood years, flood decades, even flood centuries. He concluded that the only way to characterize this temporal wiggliness was to plot the amount of fluctuation at each scale and use the slope of this line. Mandelbrot realized that this was equivalent to the kind of scaling measure that had been used for Cantor's pathological curves. As he began to apply computer graphics (figs. 1.6, 1.7), he found that these shapes were not pathological at all, but rather very common throughout the natural world. Mountain ranges had peaks within peaks, trees had branches made of branches, clouds were puffs within puffs—even his own body was full of recursive scaling structures.

The fractal simulations for natural objects in figure 1.7 were created just like the Cantor set, Koch curve, and other examples we have already seen, with a seed shape that undergoes recursive replacement. The only difference is that some of these simulations require that certain lines in the seed shape do not get

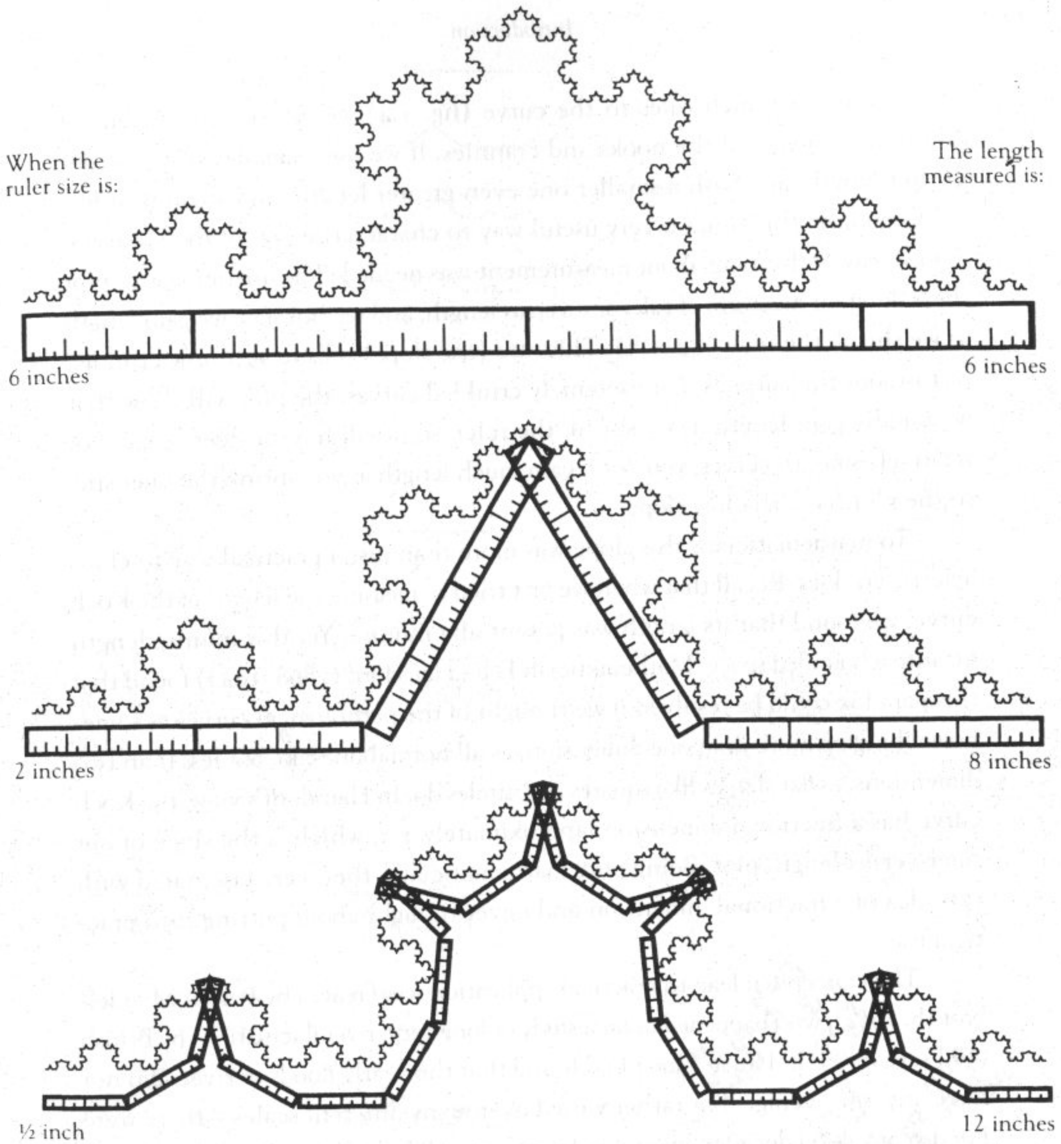


FIGURE 1.4

Measuring the length of fractal curves

The new curves of Cantor, Koch, and others represented a problem in measurement theory. The length of the curve depends on the size of the ruler. As we shrink the ruler down, the length approaches infinity.

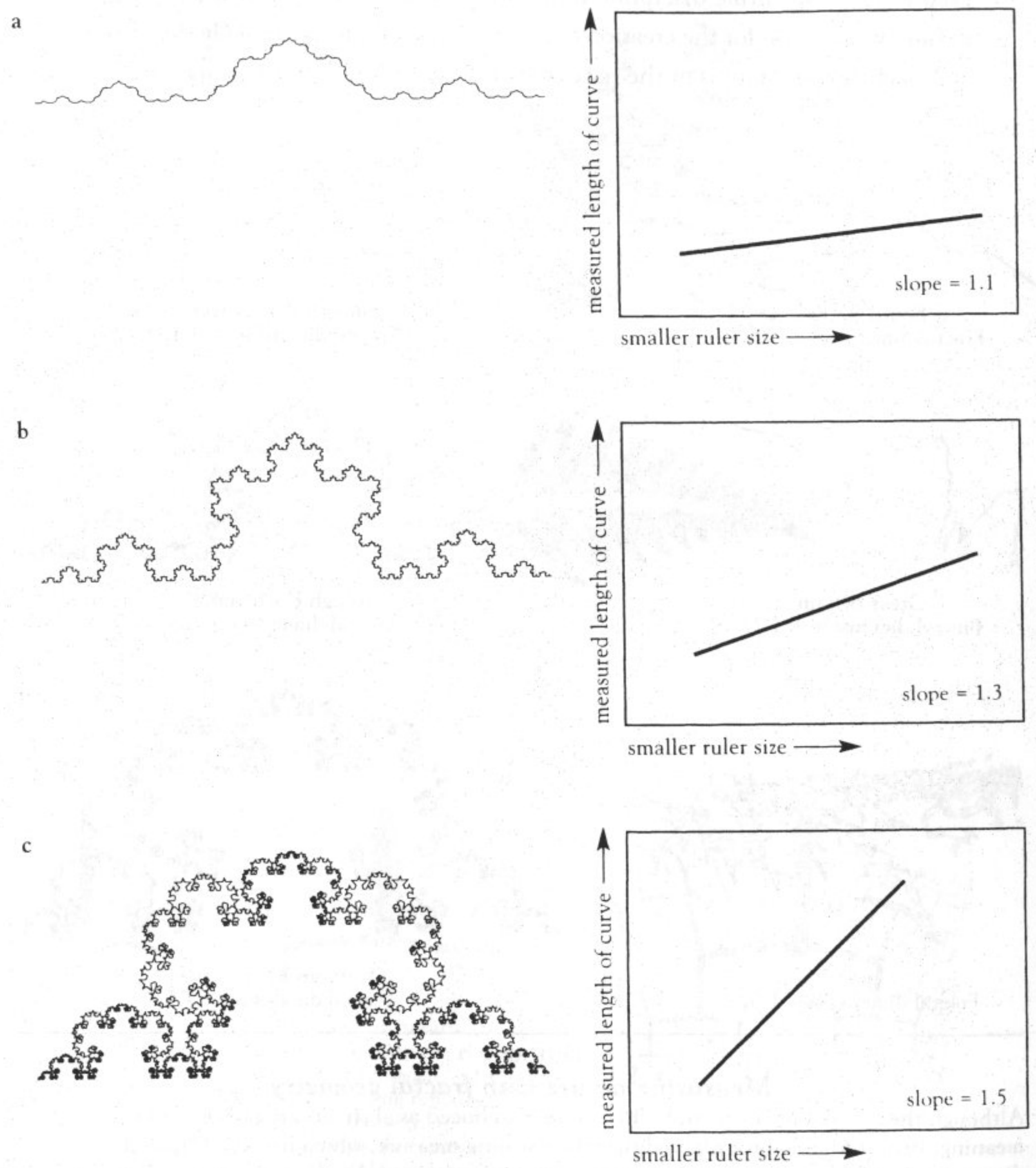


FIGURE 1.5

A better way to measure fractal curves

Our experiment in shrinking rulers wasn't a total waste. In fact, it turns out that if you keep track of how the measured length changes with ruler size, you get a very good way of characterizing the curve. A relatively smooth fractal won't increase length very quickly with shrinking ruler size, but very crinkled fractals will. **(a)** This smooth Koch curve doesn't add much length with shrinking ruler size, so the plot shows only a small rise. **(b)** Since a small ruler can get into all the nooks and crannies, this more crinkled Koch curve shows a steeper rise in measured length with a shrinking ruler. **(c)** An extremely tortuous Koch curve has a very steep slope for its plot.

Note for math sticklers: These figures are plotted on a logarithmic graph.

replaced. This is illustrated for the lung model at the bottom of figure 1.7. The lines that get replaced in each iteration are called "active lines." Those that do not get replaced are called "passive lines." We will be using the distinction between active and passive lines in simulations for African designs as well.

Mandelbrot coined the term "fractal" for this new geometry, and it is now used in every scientific discipline from astrophysics to zoology. It is one of the most powerful tools for the creation of new technologies as well as a revolutionary approach to the analysis of the natural world. In medicine, for example, fractal

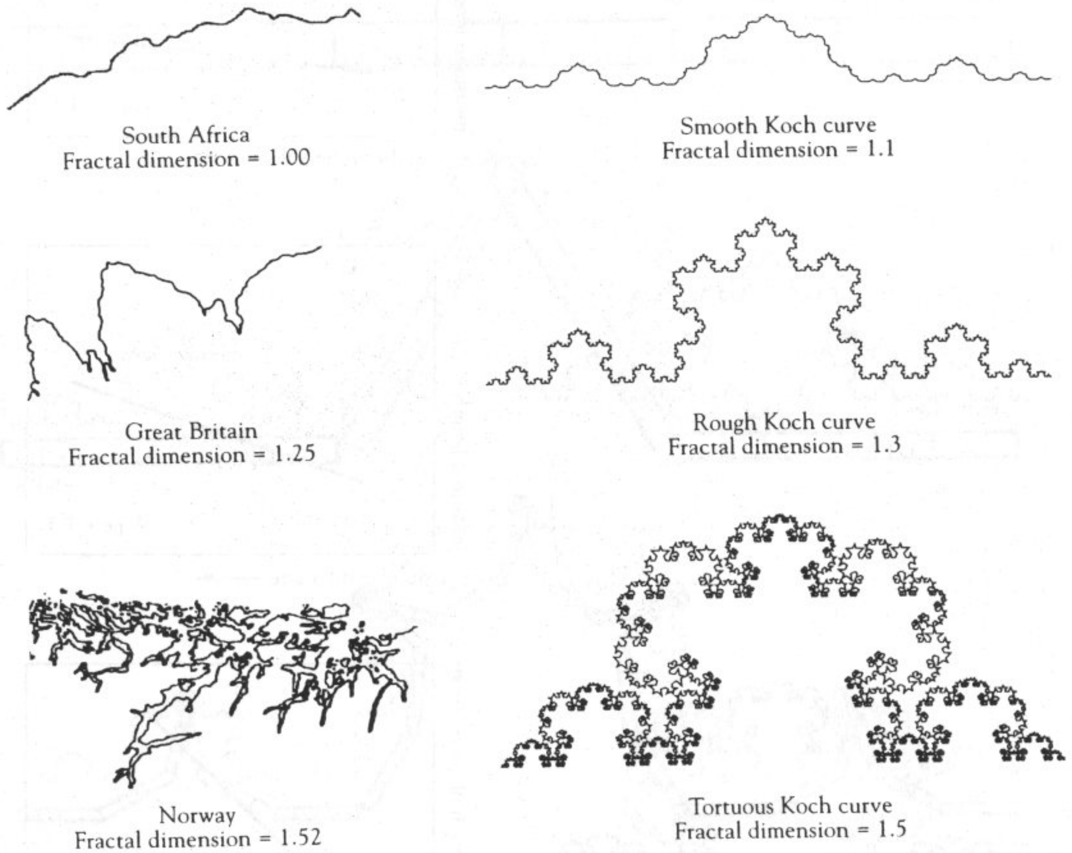


FIGURE 1.6

Measuring nature with fractal geometry

Although the curves of Cantor and others were introduced as abstractions without physical meaning, Benoit Mandelbrot realized that their scaling measure, which he called "fractal dimension," could be put to practical use in characterizing irregular shapes in nature. The classic example is the measurement of coastlines. Even though it is a very crude model, we can see how the variations of the roughness in the Koch curve are similar to the variations in these coasts. Note that the fractal dimension is our plot slope from figure 1.5; the coastlines were measured in the same way.

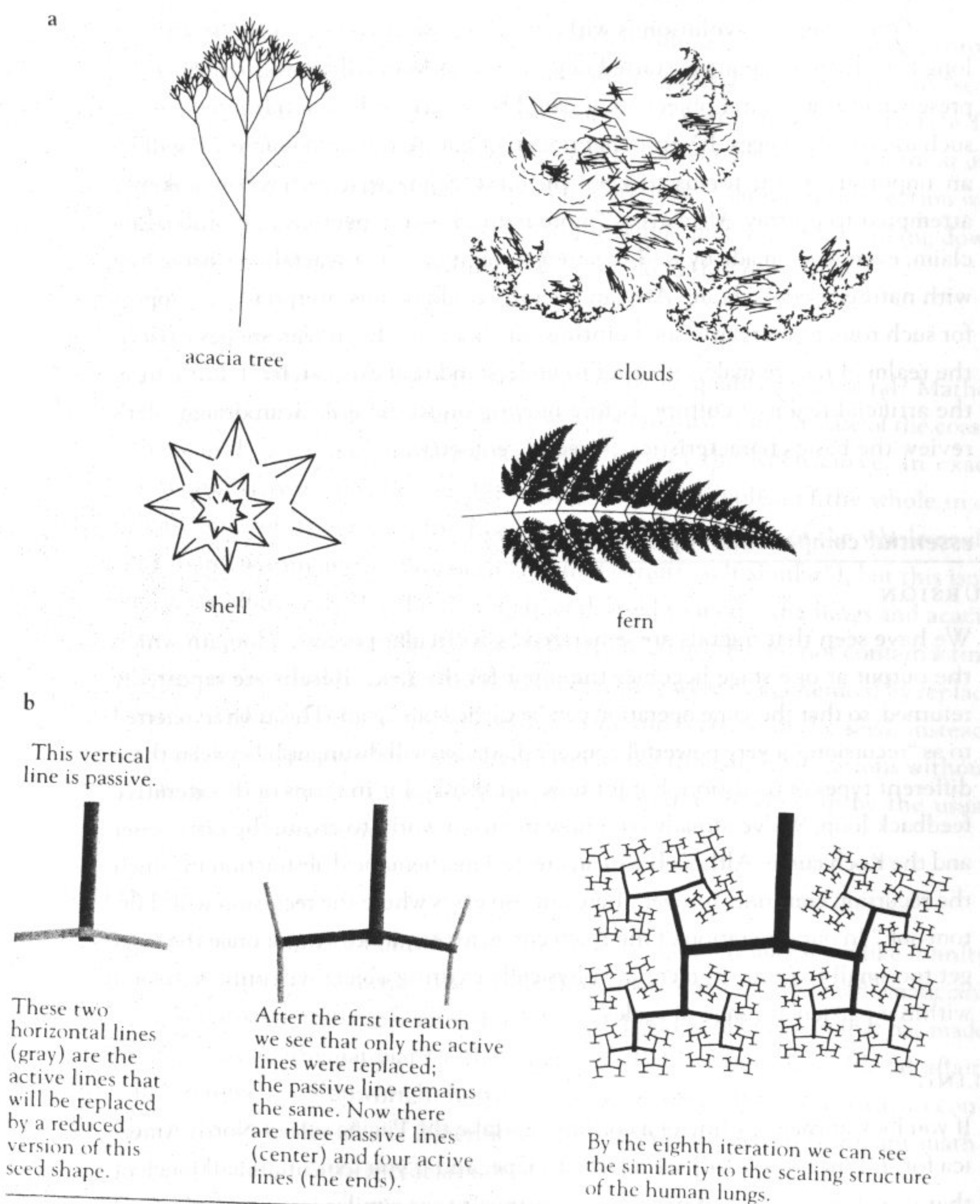


FIGURE 1.7

Simulating nature with fractal geometry

In his experiments with computer graphics, Mandelbrot found that fractal shapes abound in nature, where continual processes such as biological growth, geological change, and atmospheric turbulence result in a wide variety of recursive scaling structures (**a**). The recursive construction of these natural shapes is basically the same as that of the other fractal shapes we have seen so far. In some examples, like the lung model (**b**), certain lines of the original seed shape do not participate in the replacement step; they are called "passive lines." The ones which do go through replacement are called "active lines." Each step is referred to as an "iteration."

dimension can be used as a diagnostic tool. A healthy lung has a high fractal dimension, but when black lung disease begins it loses some of the fine branching—a condition that can be detected by measuring the fractal dimension of the X ray. For this reason, Benoit Mandelbrot was recently named an honorary member of the French Coal Miners Union.

Of course, no revolution is without its counterrevolutionaries. It was not long before some scientists started objecting that Mandelbrot was ignoring the presence of the natural objects that could be described by Euclidean geometry, such as crystals or eggs. It's true that not all of nature is fractal—and this will be an important point for us to keep in mind. Some writers have mistakenly attempted to portray Africans as "more natural"—a dangerous and misleading claim, even when made by well-meaning romantics. Since fractals are associated with nature, a book about "African fractals" could be misinterpreted as support for such romantic organicists. Pointing out that some Euclidean shapes exist in the realm of nature makes it easier to understand that African fractals are from the artificial realm of culture. Before moving on to these African designs, let's review the basic characteristics of fractal geometry.

Five essential components of fractal geometry

RECURSION

We have seen that fractals are generated by a circular process, a loop in which the output at one stage becomes the input for the next. Results are repeatedly returned, so that the same operation can be carried out again. This is often referred to as "recursion," a very powerful concept. Later we will distinguish between three different types of recursion, but for now just think of it in terms of this iterative feedback loop. We've already seen how iteration works to create the Cantor set and the Koch curve. Although we can create a mathematical abstraction in which the recursion continues forever, there are also cases where the recursion will "bottom out." In our generation of the Koch curve, for example, we quit once the lines get too small to print. In fact, any physically existing object will only be fractal within a particular range of scales.

SCALING

If you look at the coastline of a continent—take the Pacific side of North America for instance—you will see a jagged shape, and if you look at a small piece of that coastline—say, California—we continue to see similar jaggedness. In fact, a similar jagged curve can be seen standing on a cliff overlooking a rocky California shore, or even standing on that shore looking at one rock. Of course, that's

only roughly similar, and it's only good for a certain range of scales, but it is astonishing to realize how well this works for many natural features. It is this "scaling" property of nature that allows fractal geometry to be so effective for modeling. To have a "scaling shape" means that there are similar patterns at different scales within the range under consideration. Enlarging a tiny section will produce a pattern that looks similar to the whole picture, and shrinking down the whole will give us something that looks like a tiny part.

SELF-SIMILARITY

Just how similar do these patterns have to be to qualify as a fractal? Mathematicians distinguish between statistical self-similarity, as in the case of the coastline, and exact self-similarity, as in the case of the Koch curve. In exact self-similarity we need to be able to show a precise replica of the whole in at least some of its parts. In the Koch curve a precise replica of the whole could be found within any section of the fractal ("strictly self-similar"), but this isn't true for all fractals. The branching fractals used to model the lungs and acacia tree (fig. 1.7), for example, have parts (e.g., the stem) that do not contain a tiny image of the whole. Unlike the Koch curve, they were not generated by replacing every line in the seed shape with a miniature version of the seed; instead, we used some passive lines that were just carried though the iterations without change, in addition to active lines that created a growing tip by the usual recursive replacement.

INFINITY

Since fractals can be limited to a finite range of scales, it may seem like infinity is just a historical artifact, at best a Holy Grail whose quest allowed mathematicians serendipitously to stumble across fractals. It is this kind of omission that has made many pure mathematicians rather nonplussed about the whole fractal affair, and in some cases downright hostile (cf. Krantz 1989). There is no way to connect fractals to the idea of dimension without using infinity, and for many mathematicians that is their crucial role.

FRACTIONAL DIMENSION

How can it be that the Koch curve, or any member of its fractal family, has infinite length in a finite boundary? We are used to thinking of dimension as only whole numbers—the one-dimensional line, the two-dimensional plane—but the theory of measurement that governs fractals allows dimensions to be fractions. Consider, for example, the increasing dimension of the Koch curves in figure 1.6. Above the top, we could go as close as we like to a one-dimensional line. Below the bottom, we could make the curve so jagged that it starts to fill in two-dimensional areas of the plane. In between, we need an in-between dimension.

Looking for fractals in African culture

As we examine African designs and knowledge systems, these five essential components will be a useful way to keep track of what does or does not match fractal geometry. Since scaling and self-similarity are descriptive characteristics, our first step will be to look for these properties in African designs. Once we establish that theme, we can ask whether or not these concepts have been intentionally applied, and start to look for the other three essential components. We will now turn to African architecture, where we find some of the clearest illustrations of indigenous self-similar designs.

CHAPTER 2

Fractals in African settlement architecture

Architecture often provides excellent examples of cultural design themes, because anything that is going to be so much a part of our lives—a structure that makes up our built environment, one in which we will live, work or play—is likely to have its design informed by our social concepts. Take religious architecture for example. Several churches have been built using a triangular floor plan to symbolize the Christian trinity; others have used a cross shape. The Roman Pantheon was divided into three vertical levels: the bottom with seven niches representing the heavenly bodies, the middle with the 12 zodiac signs, and on top a hemisphere symbolizing the order of the cosmos as a whole.[1] But we don't need to look to grandiose monuments; even the most mundane shack will involve geometric decisions—should it be square or oblong? pitched roof or flat? face north or west?—and so culture will play a role here as well.

At first glance African architecture might seem so varied that one would conclude its structures have nothing in common. Although there is great diversity among the many cultures of Africa, examples of fractal architecture can be found in every corner of the African continent. Not all architecture in Africa is fractal—fractal geometry is not the only mathematics used in Africa—but its repeated presence among such a wide variety of shapes is quite striking.

In each case presented here we will compare the aerial photo or architectural diagram of a settlement to a computer-generated fractal model. The fractal simulation will make the self-similar aspects of the physical structure more evident, and in some cases it will even help us understand the local cultural meaning of the architecture. Since the African designers used techniques like iteration in building these structures, our virtual construction through fractal graphics will give us a chance to see how the patterns emerge through this process.

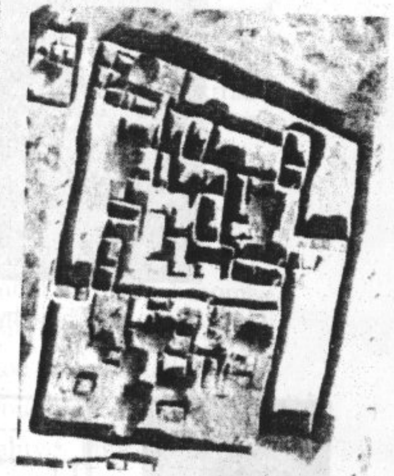
Rectangular fractals in settlement architecture

If you fly over the northern part of Cameroon, heading toward Lake Chad along the Logone River, you will see something like figure 2.1a. This aerial photo shows the city of Logone-Birni in Cameroon. The Kotoko people, who founded this city centuries ago, use the local clay to create huge rectangular building complexes. The largest of these buildings, in the upper center of the photo, is the palace of the chief, or "Miarre" (fig. 2.1b). Each complex is created by a process often called "architecture by accretion," in this case adding rectangular enclosures to preexisting rectangles. Since new enclosures often incorporate the walls of two or more of the old ones, enclosures tend to get larger and larger as you go outward from the center. The end result is the complex of rectangles within rectangles within rectangles that we see in the photo.

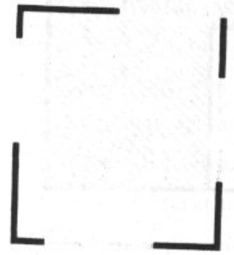
Since this architecture can be described in terms of self-similar scaling—it makes use of the same pattern at several different scales—it is easy to simulate using a computer-generated fractal, as we see in figures 2.1c–e. The seed shape of the model is a rectangle, but each side is made up of both active lines (gray) and passive lines (black). After the first iteration we see how a small version of the original rectangle is reproduced by each of the active lines. One more iteration gives a range of scales that is about the same as that of the palace; this is enlarged in figure 2.1e.

During my visit to Logone-Birni in the summer of 1993, the Miarre kindly allowed me to climb onto the palace roof and take the photo shown in figure 2.1f. I asked several of the Kotoko men about the variation in scale of their architecture. They explained it in terms of a combination of patrilocal household expansion, and the historic need for defense. "A man would like his sons to live next to him," they said, "and so we build by adding walls to the father's house." In the past, invasions by northern marauders were common, and so a larger defensive wall was also needed. Sometimes the assembly of families would outgrow this defensive enclosure, and so they would turn that wall into housing, and build an even larger enclosure around it. These scaling additions created the tradition of self-similar shapes we still see today, although the population is far below the

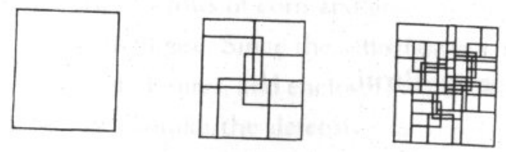
a. An aerial view of the city of Logone-Birni in Cameroon. The largest building complex, in the center, is the palace of the chief.
Photo courtesy Musée de l'Homme, Paris.



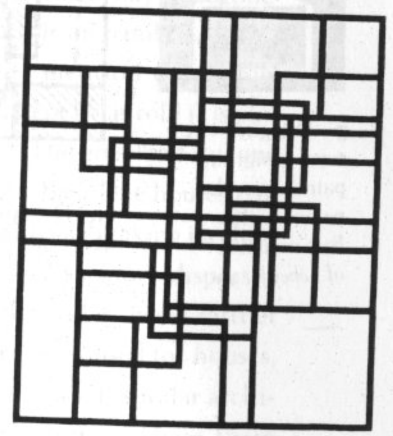
b. A closer view of the palace. The smallest rectangles, in the center, are the royal chambers.



c. Seed shape for the fractal simulation of the palace. The active lines, in gray, will be replaced by a scaled-down replica of the entire seed.



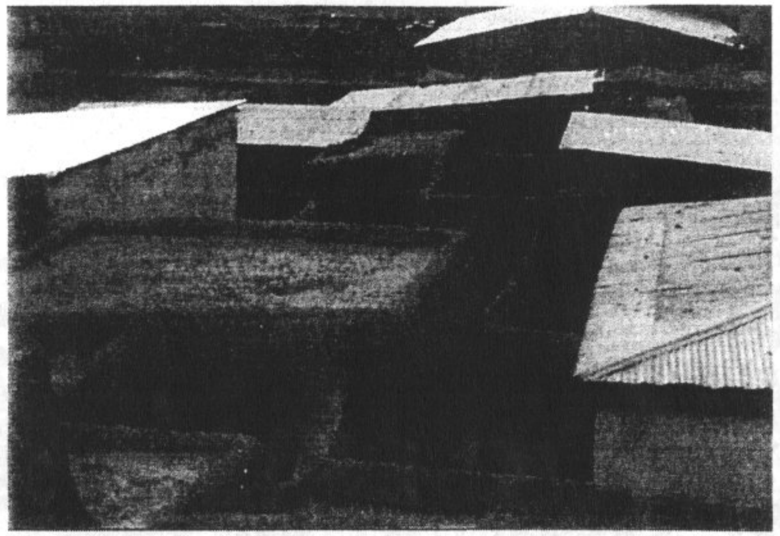
d. First three iterations of the fractal simulation.



e. Enlargment of the third iteration.

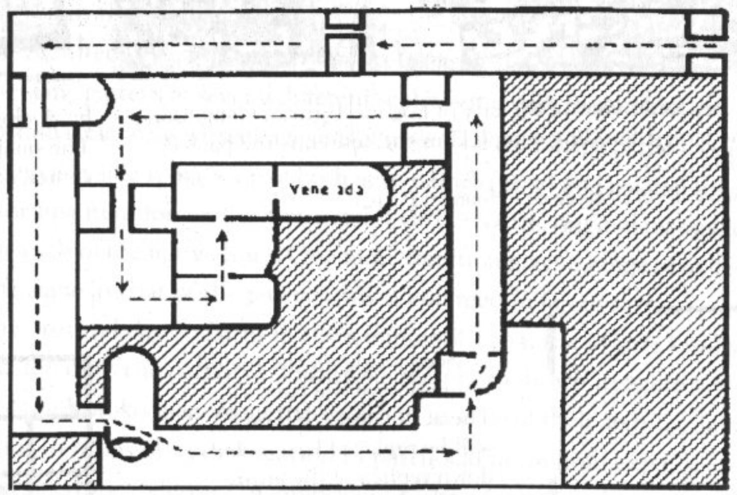
FIGURE 2.1
Logone-Birni

(figure continues)



f. Photo by the author taken from the roof of the palace.

g. The guti, the royal insignia, painted on the palace walls. *By permission of Lebeuf 1969.*



Le chemin de la lumière

h. The spiral path taken by visitors to the throne. *By permission of Lebeuf 1969.*

FIGURE 2.1 *(continued)*

Inside Logone-Birni

original 180,000 estimated for Logone-Birni's peak in the nineteenth century. At that time there was a gigantic wall, about 10 feet thick, that enclosed the perimeter of the entire settlement.

The women I spoke with were much less interested in either patrilineage or military history; their responses concerning architectural scaling were primarily about the contrast between the raw exterior walls and the stunning waterproof finish they created for courtyards and interior rooms. This began by smoothing wet walls flat with special stones, applying a resin created from a plant extract, and then adding beautifully austere decorative lines.

The most important of these decorative drawings is the *guti*, a royal insignia (fig. 2.1g). The central motif of the guti shows a rectangle inside a rectangle inside a rectangle; it is a kind of abstract model that the Kotoko themselves have created. The reason for choosing scaling rectangles as a symbol of royalty becomes clear when we look at the passage that one must take to visit the Miarre (fig. 2.1h). The passage as a whole is a rectangular spiral. Each time you enter a smaller scale, you are required to behave more politely. By the time you arrive at the throne you are shoeless and speak with a very cultured formality.[2] Thus the fractal scaling of the architecture is not simply the result of unconscious social dynamics; it is a subject of abstract representation, and even a practical technique applied to social ranking.

To the west near the Nigerian border the landscape of Cameroon becomes much greener; this is the fertile high grasslands region of the Bamileke. They too have a fractal settlement architecture based on rectangles (fig. 2.2a), but it has no cultural relation to that of the Kotoko. Rather than the thick clay of Logone-Birni, these houses and the attached enclosures are built from bamboo, which is very strong and widely available. And there was no mention of kinship, defense, or politics when I asked about the architecture; here I was told it is patterns of agricultural production that underlie the scaling. The grassland soil and climate are excellent for farming, and the gardens near the Bamileke houses typically grow a dozen different plants all in a single space, with each taking its characteristic vertical place. But this is labor intensive, and so more dispersed plantings—rows of corn and ground-nut—are used in the wider spaces farther from the house. Since the same bamboo mesh construction is used for houses, house enclosures, and enclosures of enclosures, the result is a self-similar architecture. Unlike the defensive labyrinth of Kotoko architecture, where there were only a few well-protected entryways, the farming activities require a lot of movement between enclosures, so at all scales we see good-sized openings. The fractal simulation in figures 2.2b,c shows how this scaling structure can be modeled using an open square as the seed shape.

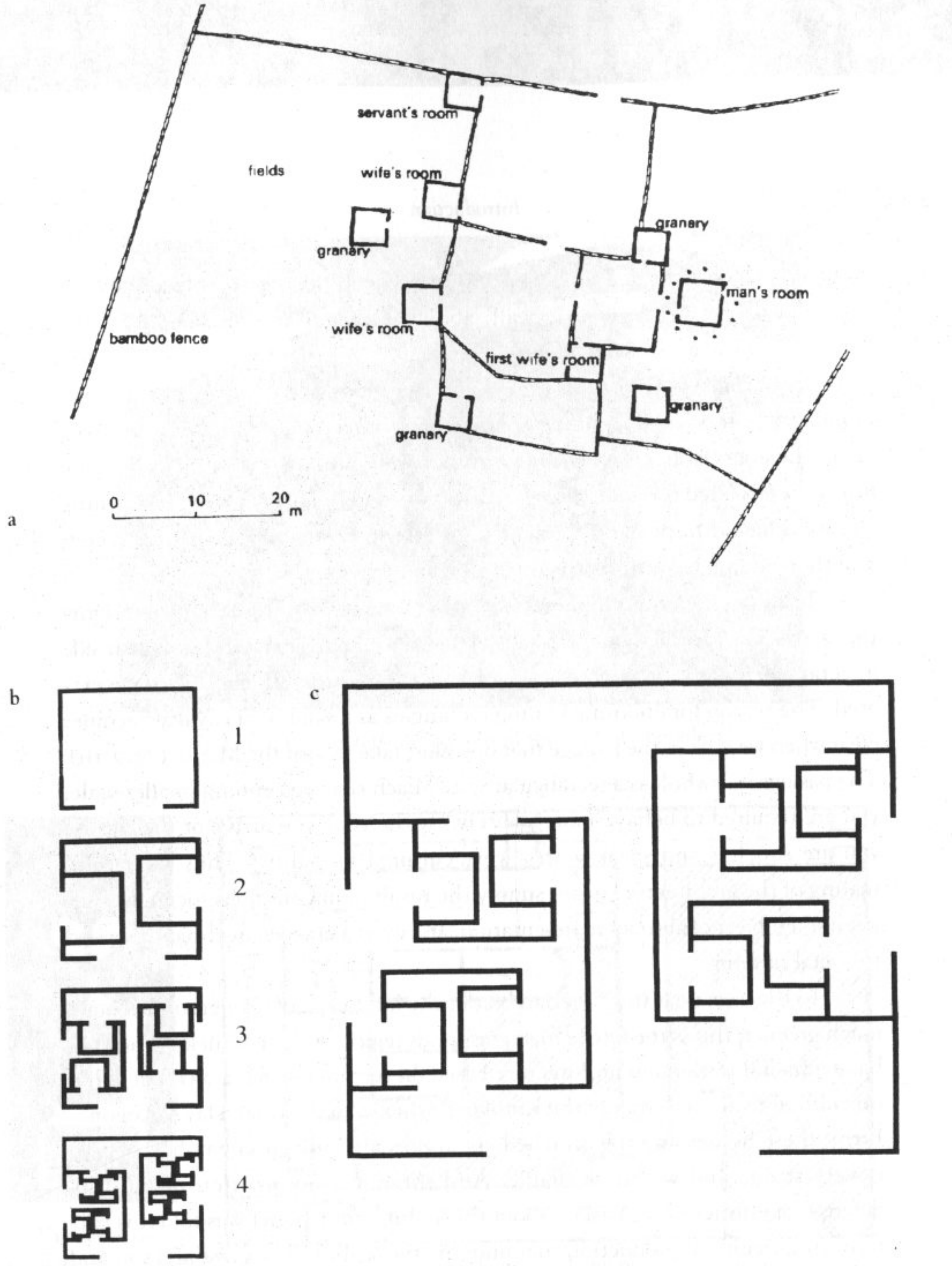


FIGURE 2.2

Bamileke settlement

(a) Plan of Bamileke settlement from about 1960. **(b)** Fractal simulation of Bamileke architecture. In the first iteration ("seed shape"), the two active lines are shown in gray. **(c)** Enlarged view of fourth iteration.

(a, *Beguin 1952; reprinted with permission from* ORSTOM).

Circular fractals in settlement architecture

Much of southern Africa is made up of arid plains where herds of cattle and other livestock are raised. Ring-shaped livestock pens, one for each extended family,[3] can be seen in the aerial photo in figure 2.3a, a Ba-ila settlement in southern Zambia. A diagram of another Ba-ila settlement (fig. 2.3d) makes these livestock enclosures ("kraals") more clear. Toward the back of each pen we find the family living quarters, and toward the front is the gated entrance for letting livestock in and out. For this reason the front entrance is associated with low status (unclean, animals), and the back end with high status (clean, people).[4] This gradient of status is reflected by the size gradient in the architecture: the front is only fencing, as we go toward the back smaller buildings (for storage) appear, and toward the very back end are the larger houses. The two geometric elements of this structure—a ring shape overall, and a status gradient increasing with size from front to back—echoes throughout every scale of the Ba-ila settlement.

The settlement as a whole has the same shape: it is a ring of rings. The settlement, like the livestock pen, has a front/back social distinction: the entrance is low status, and the back end is high status. At the settlement entrance there are no family enclosures at all for the first 20 yards or so, but the farther back we go, the larger the family enclosures become.

At the back end of the interior of the settlement, we see a smaller detached ring of houses, like a settlement within the settlement. This is the chief's extended family. At the back of the interior of the chief's extended family ring, the chief has his own house. And if we were to view a single house from above, we would see that it is a ring with a special place at the back of the interior: the household altar.

Since we have a similar structure at all scales, this architecture is easy to model with fractals. Figure 2.3b shows the first three iterations. We begin with a seed shape that could be the overhead view of a single house. This is created by active lines that make up the ring-shaped walls, as well as an active line at the position of the altar at the back of the interior. The only passive lines are those adjacent to the entrance. In the next iteration, we have a shape that could be the overhead view of a family enclosure. At the entrance to the family enclosure we have only fencing, but as we go toward the back we have buildings of increasing size. Since the seed shape used only passive lines near the entrance and increasingly larger lines toward the back, this iteration of our simulation has the same size gradient that the real family enclosure shows. Finally, the third iteration provides a structure that could be the overhead view of the whole settlement. At the entrance to the settlement we have only fencing, but as we go toward

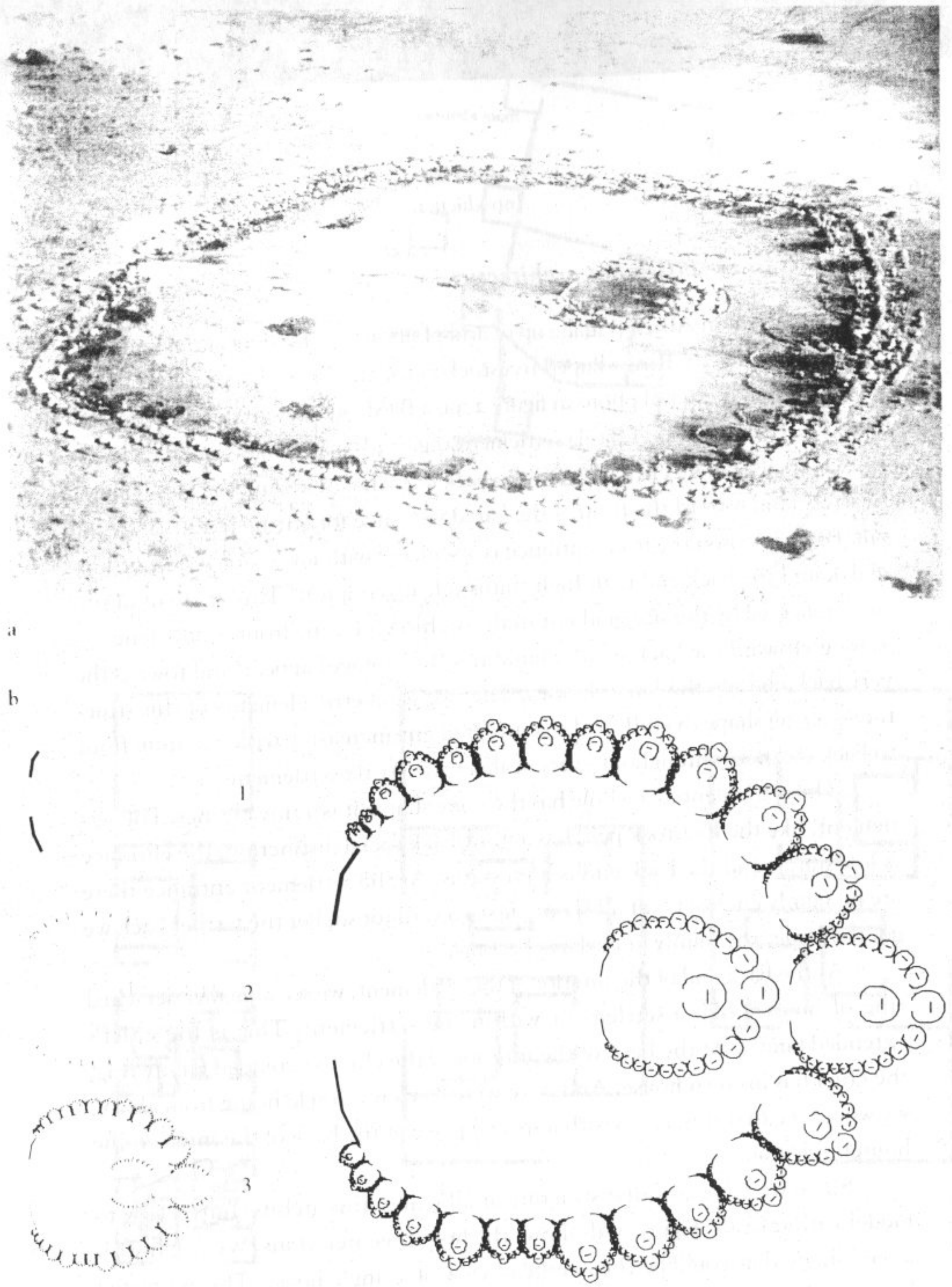


FIGURE 2.3
Ba-ila
(a) Aerial photo of Ba-ila settlement before 1944. **(b)** Fractal generation of Ba-ila simulation. Note that the seed shape has only active lines (gray) except for those near the opening (black). (a, *American Geographic Institute.*)

the back we have enclosures of increasing size. Again, by having the seed shape use only passive lines near the entrance and increasingly larger lines toward the back, this iteration of our simulation has the same size gradient that the real settlement shows.

I never visited the Ba-ila myself; most of my information comes from the classic ethnography by Edwin Smith and Andrew Dale, published in 1920. While their colonial and missionary motivations do not inspire much trust, they often showed a strong commitment toward understanding the Ba-ila point of view for social structure. Their analysis of Ba-ila settlement architecture points out fractal attributes. They too noted the scaling of house size, from those less than 12 feet wide near the entrance, to houses more than 40 feet wide at the back, and explained it as a social status gradient; "there being a world of difference between the small hovel of a careless nobody and the spacious dwelling of a chief" (Smith and Dale 1968, 114).

It is in Smith's discussion of religious beliefs, however, that the most striking feature of the Ba-ila's fractal architecture is illuminated. Unlike most missionaries of his time, Smith was a strong proponent of respect for local religions. He was no relativist—understanding and respect were strategies for conversion—but his delight in the indigenous spiritual strength comes across clearly in his writings and provided him with insight into the subtle relation of the social, sacred, and physical structure of the Ba-ila architectural plan.

> In this village there are about 250 huts, built mostly on the edge of a circle four hundred yards in diameter. Inside this circle there is a subsidiary one occupied by the chief, his family, and cattle. It is a village in itself, and the form of it in the plan is the form of the greater number of Ba-ila villages, which do not attain to the dimensions of Shaloba's capital. The open space in the center of the village is also broken by a second subsidiary village, in which reside important members of the chief's family, and also by three or four miniature huts surrounded by a fence: these are the *manda a mizhimo* ("the manes' huts") where offerings are made to the ancestral spirits. Thus early do we see traces of the all-pervading religious consciousness of the Ba-ila. (Smith and Dale 1968, 113)

In the first iteration of the computer-generated model there is a detached active line inside the ring, at the end opposite the entrance. This was motivated by the ring comprising the chief's family, but it also describes the location of the sacred altar within each house. As a logician would put it, the chief's family ring is to the whole settlement as the altar is to the house. It is not a status gradient, as we saw with the front-back axis, but rather a recurring functional role between different scales: "The word applied to the chief's relation to his people is *kulela:* in the extracts given above we translate it 'to rule,' but it has this only as a secondary meaning. Kulela is primarily to nurse, to cherish; it is the word applied to a woman caring for her child. The chief is the father of the community; they are his children, and what he does is *lela* them" (Smith and Dale 1968, 307).

This relationship is echoed throughout family and spiritual ties at all scales, and is structurally mapped through the self-similar architecture. The nesting of circular shapes—ancestral miniatures to chief's house ring to chief's extended family ring to the great outer ring—was not a status gradient, as we saw for the enclosure variation from front to back, but successive iterations of *lela*.

A very different circular fractal architecture can be seen in the famous stone buildings in the Mandara Mountains of Cameroon. The various ethnic groups of this area have their own separate names, but collectively are often referred to as Kirdi, the Fulani word for "pagan," because of their strong resistance against conversion to Islam. Their buildings are created from the stone rubble that commonly covers the Mandara mountain terrain. Much of the stone has natural fracture lines that tend to split into thick flat sheets, so these ready-made bricks—along with defensive needs—helped to inspire the construction of their huge castlelike complexes. But rather than being the Euclidean shapes of European castles, this African architecture is fractal.

Figure 2.4a shows the building complex of the chief of Mokoulek, one of the Mofou settlements. An architectural diagram of Mokoulek, drawn by French researchers from the ORSTOM science institute, shows its overall structure (fig. 2.4b). It is primarily composed of three stone enclosures (the large circles), each of which surrounds tightly spiraled granaries (small circles). The seed shape for the simulation requires a circle, made of passive lines, and two different sets of active lines (fig. 2.4c). Inside the circle is a scaling sequence of small active lines; these will become the granaries. Outside the circle there is a large active line; this will replicate the enclosure filled with granaries. By the fourth iteration we have created three enclosures filled with spiral clusters of granaries, plus one unfilled. The real diagram of Mokoulek shows several unfilled circles—evidence that not everything in the architectural structure can be accounted for by fractals. Nevertheless, an important feature is suggested by the simulation.

In the first iteration we see that the large external active line is to the left of the circle. But since it is at an angle, the next iteration finds this active line above and to the right. If we follow the iterations, we can see that the *dynamic construction* of the complex has a spiral pattern; the replications whorl about a central location. This spiral dynamic can be missed with just a static view—I certainly didn't see it before I tried the simulation—but our participation in the virtual construction makes the spiral quite evident.[5] The similarity between the small spirals of granaries inside the enclosures and this large-scale spiral shape of the

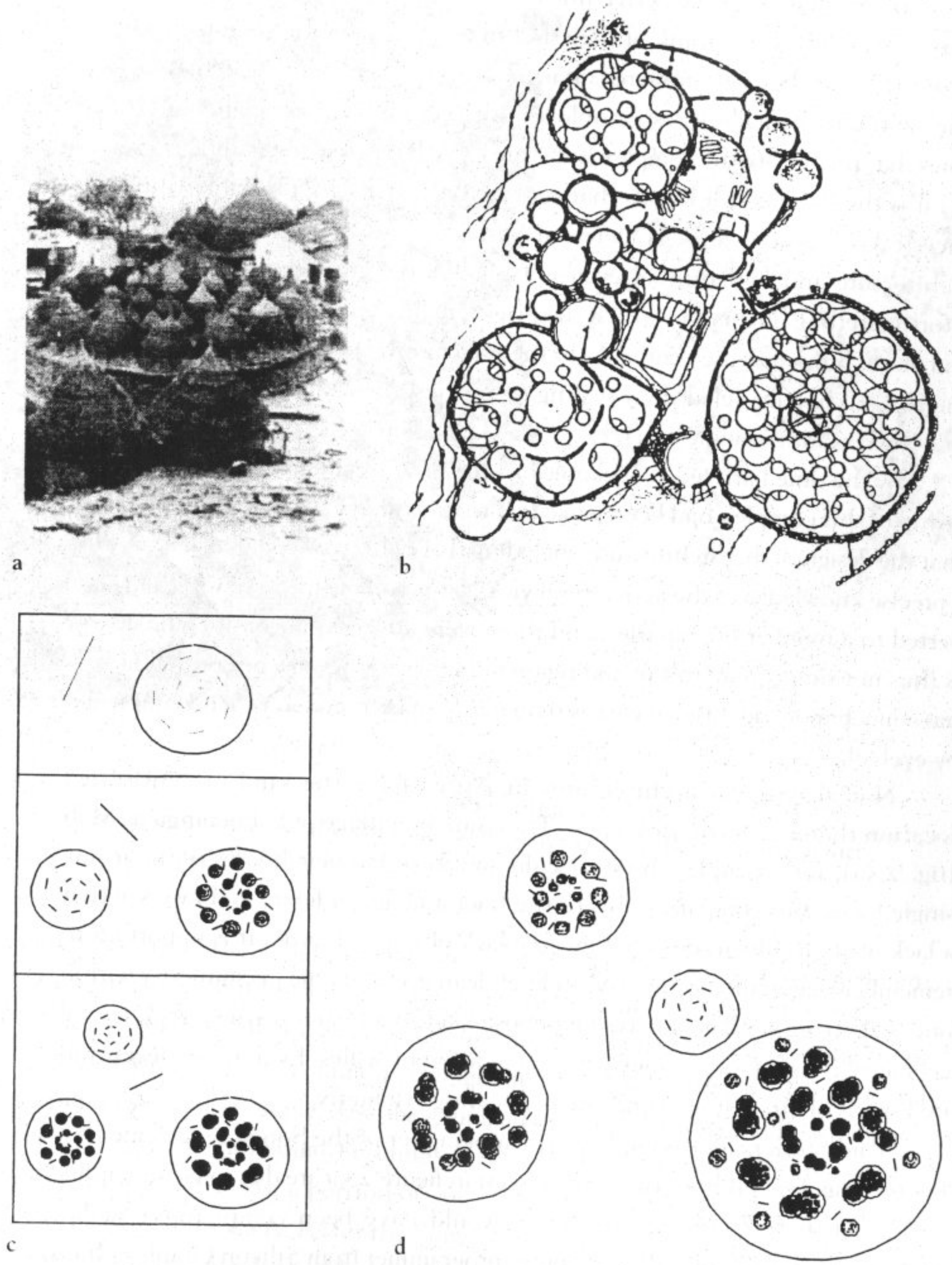


FIGURE 2.4
Mokoulek

(a) Mokoulek, Cameroon. The small buildings inside the stone wall are granaries. The rectangular building (top right) holds the sacred altar. **(b)** Architectural diagram of Mokoulek. **(c)** First three iterations of the Mokoulek simulation. The seed shape is composed of a circle drawn with passive lines (black) and with gray active lines both inside and outside the circle. **(d)** Fourth iteration of the Mokoulek simulation.

(a and b, by permission from Seignobos 1982.)

complex as a whole indicates that the fractal appearance of the architecture is not merely due to a random accumulation of various-sized circular forms. The idea of circles of increasing size, spiraling from a central point, has been applied at two different scales, and this structural coherence is confirmed by the architects' own concepts.

In our simulation the active line became located toward the center of the spiral. The Mofou also think of their architecture as spiraling from this central location, which holds their sacred altar. The altar is a kind of conceptual "active line" in their schema; it is responsible for the iterations of life. Seignobos (1982) notes that this area of the complex is the site of both religious and political authority; it is the location for rituals that generate cycles of agricultural fertility and ancestral succession. This geometric mapping between the scaling circles of the architecture and the spiritual cycles of life is represented in their divination ("fortunetelling") ritual, in which the priest creates concentric circles of stones and places himself at the center. As in the *guti* painting in Logone-Birni, in which the Kotoko had modeled their scaling rectangles, the Mofou have also created their own scaling simulation.

By the time I arrived at Mokoulek in 1994 the chief had died, and the ownership of this complex had been passed on to his widows. The new chief told me that the design of this architecture, including that of his new complex, began with a precise knowledge of the agricultural yield. This volume measure was then converted to a number of granaries, and these were arranged in spirals. The design is thus not simply a matter of adding on granaries as they are needed; in fact, it has a much more quantitative basis than my computer model, which I simply did by eyeball.

Not all circular architectures in Africa have the kind of centralized location that we saw in Mokoulek. The Songhai village of Labbezanga in Mali (fig. 2.5a), for example, shows circular swirls of circular houses without any single focus. But comparing this to the fractal image of figure 2.5b, we see that a lack of central focus does not mean a lack of self-similarity. It is important to remember that while "symmetry" in Euclidean geometry means similarity within one scale (e.g., similarity between opposite sides in bilateral symmetry), fractal geometry is based on symmetry between different scales. Even these decentralized swirls of circular buildings show a scaling symmetry.

Paul Stoller, an accomplished ethnographer of the Songhai, tells me that the rectangular buildings that can be seen in figure 2.5a are due to Islamic influence, and that the original structure would have been completely circular. Thanks to Peter Broadwell, a computer programmer from Silicon Graphics Inc., we were able to run a quantitative test of the photo that confirmed what our eyes

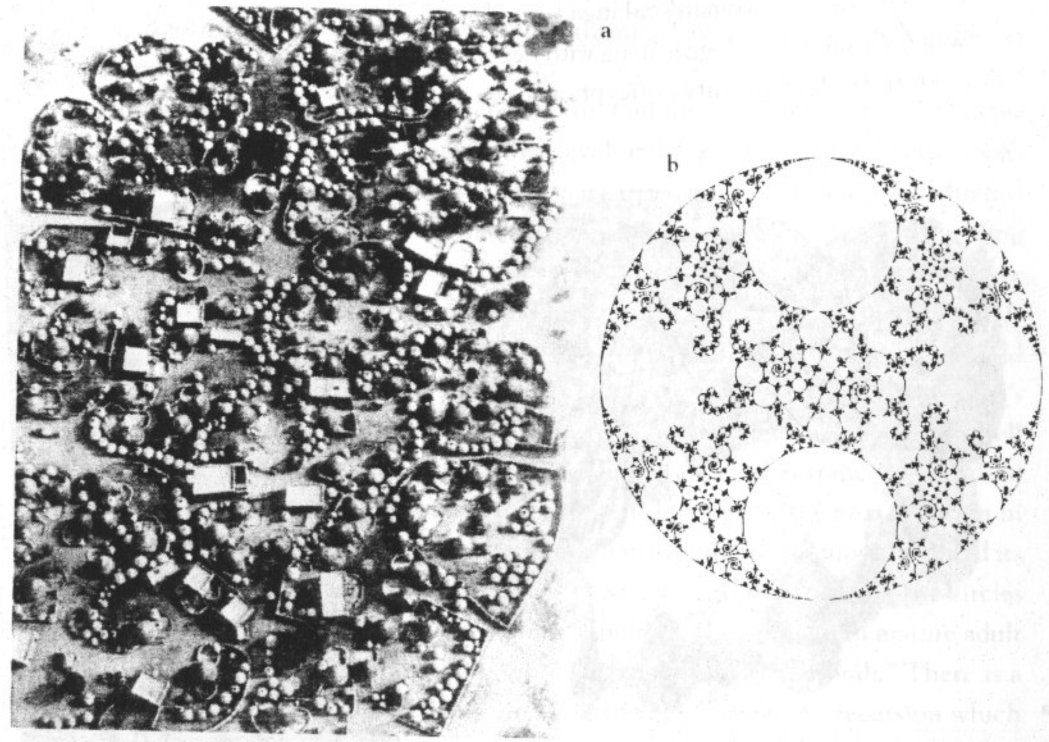


FIGURE 2.5
Labbezanga
(a) Aerial view of the village of Labbezanga in Mali. **(b)** Fractal graphic.
(**a**, *photo by Georg Gerster;* **b**, *by permission of Benoit Mandelbrot.*)

were telling us: the Songhai architecture can be characterized by a fractal dimension similar to that of the computer-generated fractal of figure 2.5b.[6]

This kind of dense circular arrangement of circles, while occurring in all sorts of variations, is common throughout inland west Africa. Bourdier and Trinh (1985), for example, describe a similar circular architecture in Burkina Faso. The scaling of individual buildings is beautifully diagrammed in their cover illustration (fig. 2.6a), a portion of one of the large building complexes created by the Nankani society. As for the Songhai, foreign cultural influences have now introduced rectangular buildings as well. In the Nankani complex the outermost enclosure (the perimeter of the complex) is socially coded as male. As we move in, the successive enclosures become more female associated, down to the circular woman's *dégo* (fig. 2.6b), the circular fireplace, and finally the scaling stacks of pots (fig. 2.6c).

Using a technique quite close to that of the Kotoko women, the women of Nankani also waterproof and decorate these walls. The recurrent image of a triangle in these decorations (see walls of dégo) represents the *zalanga*, a nested stack of calabashes (circular bowls carved from gourds) that each woman keeps in her kitchen (fig. 2.6d). Since these calabashes are stacked from large to small, they (and the rope that holds them) form a triangle—thus the triangular decorations also represent scaling circles, just in a more abstract way. The smallest container in a woman's zalanga is the *kumpio*, which is a shrine for her soul. When she dies, the zalanga, along with her pots, is smashed, and her soul is released to eternity. The eternity concept, associated with well-being, is symbolically

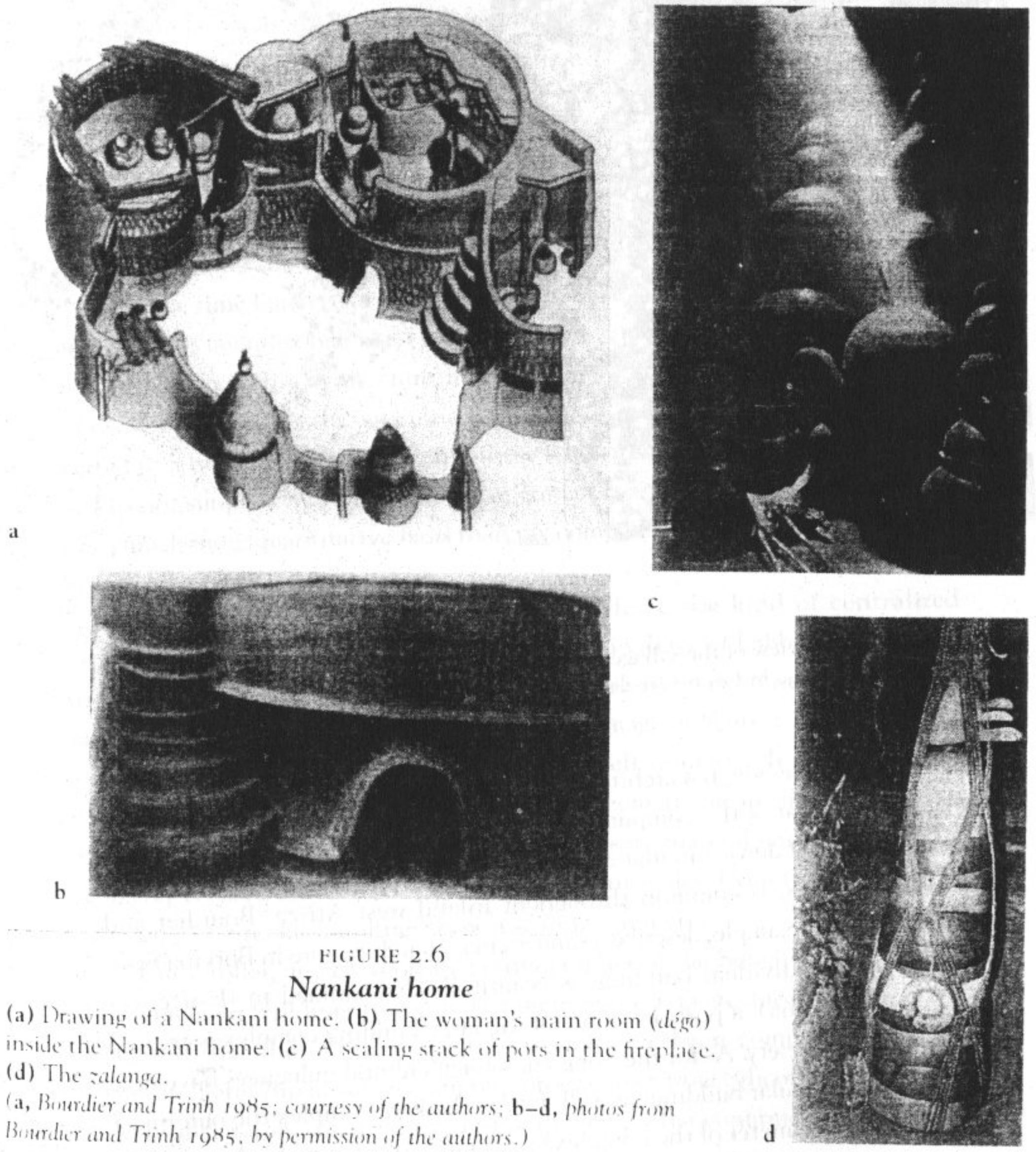


FIGURE 2.6

Nankani home

(a) Drawing of a Nankani home. **(b)** The woman's main room (*dégo*) inside the Nankani home. **(c)** A scaling stack of pots in the fireplace. **(d)** The *zalanga*.

(*a, Bourdier and Trinh 1985; courtesy of the authors;* **b–d,** *photos from Bourdier and Trinh 1985, by permission of the authors.*)

represented by the coils of a serpent of infinite length, sculpted into the walls of these homes.

From the 20-meter diameter of the building complex to the 0.2-meter kumpio—and not simply at one or two levels in between, but with dozens of self-similar scalings—the Nankani fractal spans three orders of magnitude, which is comparable to the resolution of most computer screens. Moreover, these scaling circles are far from unconscious accident: as in several other architectures we have examined, they have made conscious use of the scaling in their social symbolism. In this case, the most prominent symbolism is that of birthing. When a child is born, for example, it must remain in the innermost enclosure of the women's dégo until it can crawl out by itself. Each successive entrance is—spatially as well as socially—a rite of passage, starting with the biological entrance of the child from the womb. It leaves each of these nested chambers as the next iteration in life's stages is born. The zalanga models the entire structure in miniature, and its destruction in the event of death maps the journey in reverse: from the circles of the largest calabash to the tiny kumpio holding the soul—from mature adult to the eternal realm of ancestors who dwell in "the earth's womb." There is a conscious scheme to the scaling circles of the Nankani: it is a recursion which bottoms-out at infinity.

Branching fractals

While African circular buildings are typically arranged in circular clusters, the paths that lead through these settlements are typically not circular. Like the bronchial passages that oxygenate the round alveoli of the lungs, the routes that nourish circular settlements often take a branching form (e.g., figure 2.7). But despite my unavoidably organicist metaphor, these cannot be simply reduced to unconscious traces of minimum effort. For one thing, conscious design criteria are evident in communities in which there is an architectural transition from circular to rectangular buildings, since they can choose to either maintain or erase the branching forms.

Discussion concerning such decisions are apparent in the settlement of Banyo, Cameroon, where the transition has a long history (Hurault 1975). I found that few circular buildings were left, but those that were still intact served as an embodiment of cultural memory. This role was honored in the case of the chief's complex and exploited in the case of a blacksmith's shop, which was the site of occasional tourist visits. After passing approval by the government officials and the sultan, I was greeted by the official city surveyor, who—considering the fact that his raison d'être was Euclideanizing the streets—showed surprising

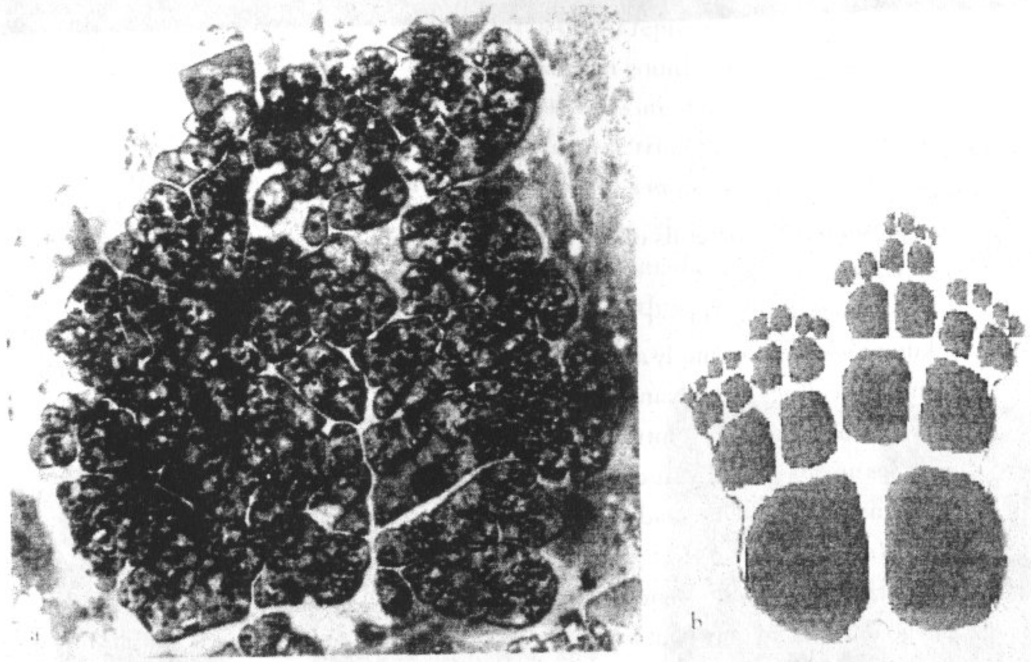


FIGURE 2.7
Branching paths in a Senegalese settlement
(a) Aerial photo of a traditional settlement in northeast Senegal. The space between enclosure walls, serving as roads and footpaths, creates a branching pattern. **(b)** A branching fractal can be created by the background of a scaling set of circular shapes.
(**a**, *courtesy Institut Geographique du Senegal.*)

appreciation for my project and helped me locate the most fractal area of the city (fig. 2.8a). At the upper left of the photo we see a portion of the Euclidean grid that covers the rest of the city, but most of this area is still fractal. The location of this carefully maintained branching—fanning out from a large plaza that is bordered by the palace of the sultan and the grand mosque—is no coincidence. By marking my position on the aerial photo as I traveled through (fig. 2.8b), I was later able to create a map by digitally altering the photo image (fig. 2.8c). This provides a stark outline—looking much like the veins in a leaf—of the fractal structure of this transportation network. I may have plunged through a wall or two in creating this map, but it certainly underestimates the fine branching of the footpaths, as I did not attempt to include their extensions into private housing enclosures.

How does the creation of these scaling branches interact with the kinds of iterative construction and social meaning we have seen associated with other examples of fractal architecture? A good illustration can be found in the

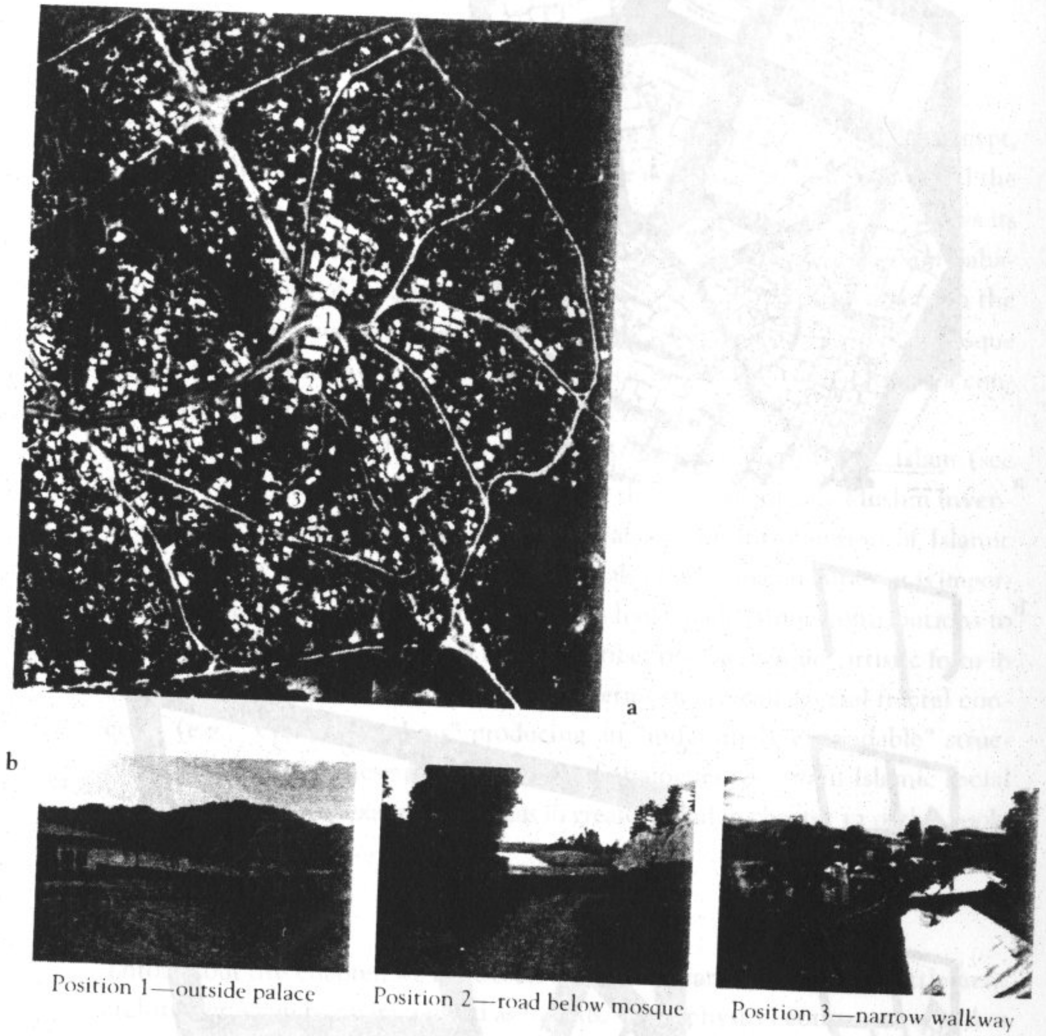
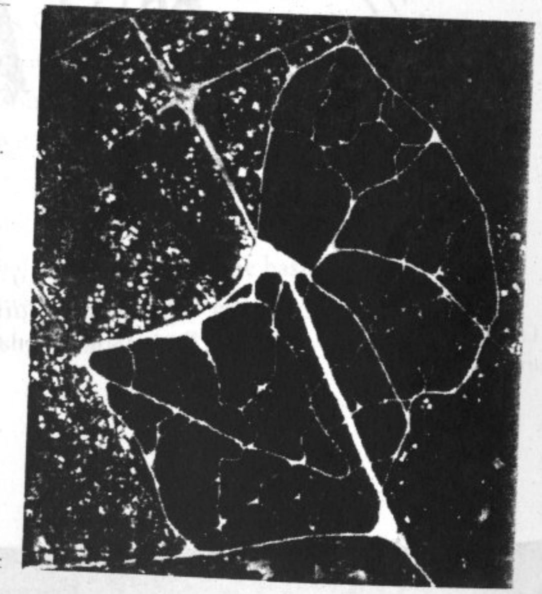


FIGURE 2.8
Branching paths in Banyo
(a) Aerial photo of the city of Banyo, Cameroon. **(b)** Successive views of the branching paths, as marked on the photo above. The clay walls require their own roof, which comes in both thatched and metal versions along the walkway in the last photo. **(c)** Aerial photo of Banyo with only public paths showing.
(**a**, *courtesy National Institute of Cartography, Cameroon.*)



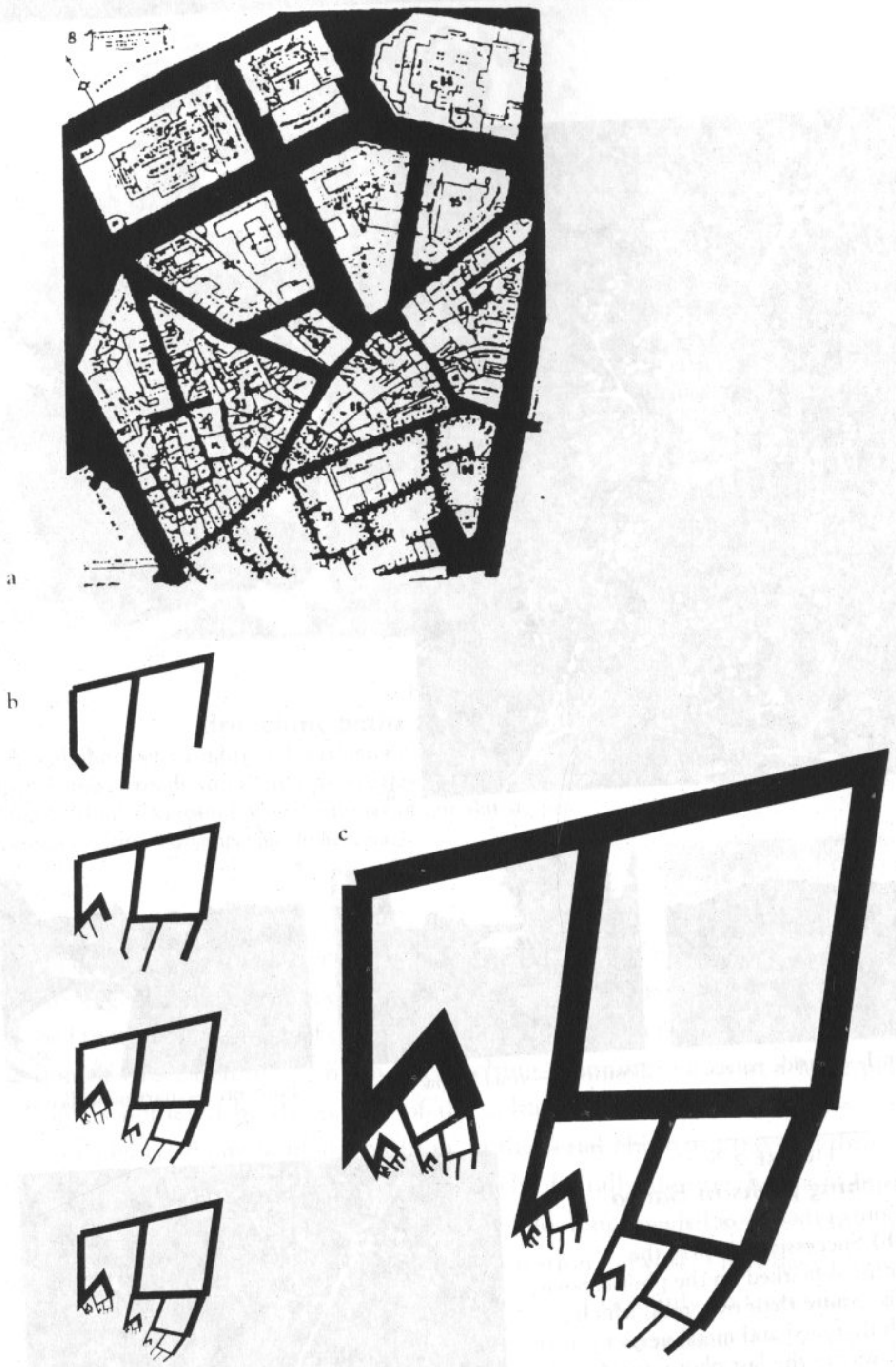


FIGURE 2.9
Streets of Cairo
(a) Map of streets of Cairo, 1898. (b) Fractal simulation for Cairo streets. (c) Enlarged view of fourth iteration.

branching streets of North African cities. Figure 2.9a shows a map of Cairo, Egypt, in 1898. The map was created by an insurance company, and I have colored the streets black to make the scaling branches more apparent. Figure 2.9b shows its computer simulation. Delaval (1974) has described the morphogenesis of Saharan cities in terms of successive additions similar to the line replacement in the fractal algorithms we have used here. The first "seed shape" consists of a mosque connected by a wide avenue to the marketplace, and successive iterations of construction add successive contractions of this form.

Since these fractal Saharan settlement architectures predate Islam (see Devisse 1983), it would be misleading to see them as an entirely Muslim invention; but given the previous observations about the introduction of Islamic architecture as an interruption of circular fractals in sub-Saharan Africa, it is important to note that Islamic cultural influences have made strong contributions to African fractals as well. Heaver (1987) describes the "arabesque" artistic form in North African architecture and design in terms that recall several fractal concepts (e.g., "cyclical rhythms" producing an "indefinitely expandable" structure). He discussed these patterns as visual analogues to certain Islamic social concepts, and we will examine his ideas in greater detail in chapter 12 of this book.

Conclusion

Throughout this chapter, we have seen that a wide variety of African settlement architectures can be characterized as fractals. Their physical construction makes use of scaling and iteration, and their self-similarity is clearly evident from comparison to fractal graphic simulations. Chapter 3 will show that fractal architecture is not simply a typical characteristic of non-Western settlements. This alone does not allow us to conclude an indigenous African knowledge of fractal geometry; in fact, I will argue in chapter 4 that certain fractal patterns in African decorative arts are merely the result of an intuitive esthetic. But as we have already seen, the fractals in African architecture are much more than that. Their design is linked to conscious knowledge systems that suggest some of the basic concepts of fractal geometry, and in later chapters we will find more explicit expressions of this indigenous mathematics in astonishing variety and form.

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CHAPTER 3

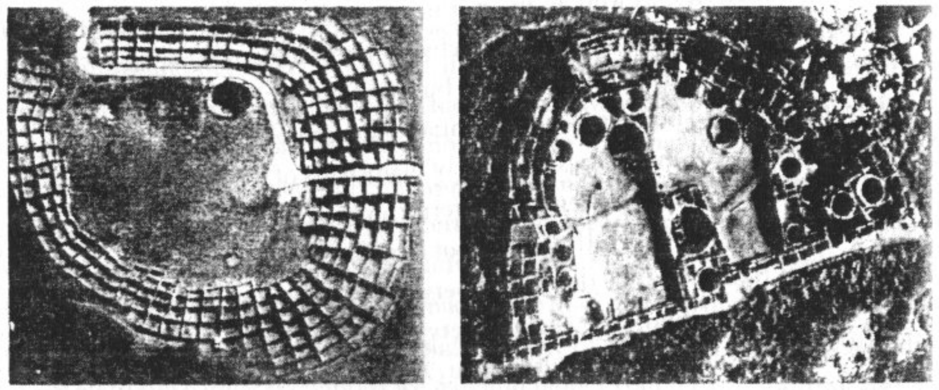
Fractals in cross-cultural comparison

The fractal settlement patterns of Africa stand in sharp contrast to the Cartesian grids of Euro-American settlements. Why the difference? One explanation could be the difference in social structure. Euro-American cultures are organized by what anthropologists would call a "state society." This includes not just the modern nation-state, but refers more generally to any society with a large political hierarchy, labor specialization, and cohesive, formal controls—what is sometimes called "top-down" organization. Precolonial African cultures included many state societies, as well as an enormous number of smaller, decentralized social groups, with little political hierarchy—that is, societies that are organized "bottom-up" rather than "top-down."[1] But if fractal architecture is simply the automatic result of a nonstate social organization, then we should see fractal settlement patterns in the indigenous societies of many parts of the world. In this chapter we will examine the settlement patterns found in the indigenous societies of the Americas and the South Pacific, but our search will turn up very few fractals. Rather than dividing the world between a Euclidean West and fractal non-West, we will find that each society makes use of its particular design themes in organizing its built environment. African architecture tends to be fractal because that is a prominent design theme in African culture. In fact, this cultural specificity of design themes is true not only for architecture, but for many other types of material design and cultural practices as well. We will begin our survey with a brief look at the design themes in Native American societies, which included both hierarchical state empires as well as smaller, decentralized tribal cultures.

Native American design

The Ancestral Pueblo society dwelled in what is now the southwestern United States around 1100 C.E. Aerial photos of these sites (fig. 3.1) are some of the most famous examples of Native American settlements. But as we can see from this vantage point, the architecture is primarily characterized by an enormous *circular* form created from smaller *rectangular* components—certainly not the same shape at two different scales. This juxtaposition of the circle and the quadrilateral (rectangle or cross-shaped) form is not a coincidence; the two forms are the most important design themes in the material culture of many Native American societies, including both North and South continents.

As far as architecture is concerned, there are no examples of the nonlinear scaling we saw in Africa. The only Native American architectures that come close are a few instances of linear concentric figures (fig. 3.2a). Shapes approximating concentric circles can be seen in the Poverty Point complex in northern Louisiana, for example, and there were concentric circles of tepees in the Cheyenne camps. The step-pyramids of Mesoamerica look like concentric squares when viewed from above. But linear concentric figures are not fractals. First, these are linear layers: the distance between lines is always the same, and thus the number of concentric circles within the largest circle is finite. The nonlinear scaling of fractals requires an ever-changing distance between lines,



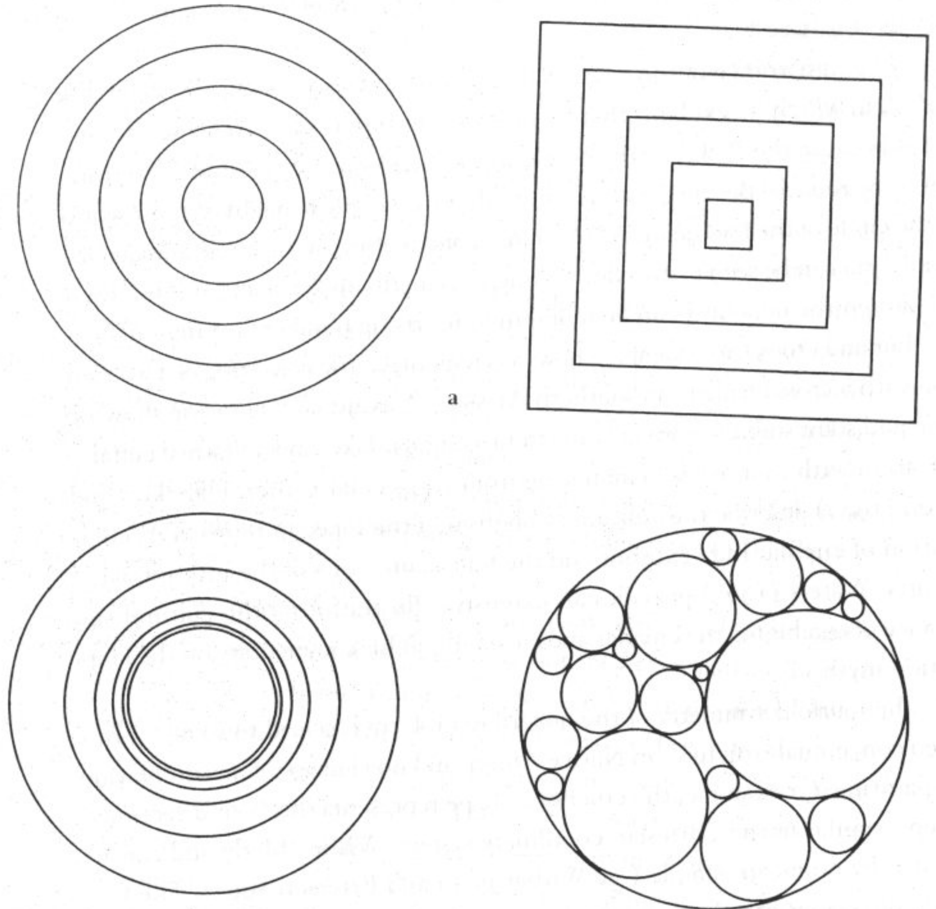
a b

FIGURE 3.1

Euclidean geometry in Native American architecture

(a) Aerial photo of Bandelier, one of the Ancestral Pueblo settlements (starting around 1100 C.E.) in northwestern New Mexico. **(b)** Aerial photo of Pueblo Bonito, another Ancestral Pueblo settlement (starting around 950 C.E.). Note that they are mostly rectangular at the smallest scale and circular at the largest scale.

*(**a,** photo by Tom Baker; **b,** photo by Georg Gerster.)*



a b c

FIGURE 3.2

Linear concentric forms in Native American architecture

(a) Native American architecture is typically based on quadrilateral grids or a combination of circular and grid forms. The only examples of scaling shapes are these linear concentric forms. In the Poverty Point complex, for example, concentric circles were used, and concentric squares can be seen if we look at the Mexican step pyramids from above. These forms are better characterized as Euclidean than fractal for two reasons: **(b)** First, they are linear. Here is an example of a nonlinear concentric circle. While the linear version must have a finite number of circles, this figure could have an infinite number and still fit in the same boundary. **(c)** Second, they only scale with respect to one point (the center). Here is an example of circles with more global scaling symmetry.

which means there can be an infinite number in a finite space (fig. 3.2b). Second, even nonlinear concentric circles are only self-similar with respect to a single locus (the center point), rather than having the global self-similarity of fractals (fig. 3.2c).

The importance of the circle is detailed in a famous passage by Black Elk (1961), in which he explains that "everything an Indian does is in a circle, and that is because the Power of the World always works in circles, and everything tries to be round." But he goes on to note that his people thought of their world as "the circle of the four quarters." A similar combination of the circle and quadrilateral form can be seen many Native American myths and artifacts; it is not their only design theme, but it can be found in a surprising number of different societies. Burland (1965), for example, shows a ceremonial rattle consisting of a wooden hoop with a cross inside from southern Alaska, a Navajo sand painting showing four equidistant stalks of corn growing from a circular lake, and a Pawnee buffalo-hide drum with four arrows emanating from its circular center. Nabokov and Easton (1989) describe the cultural symbolism of the tepee in terms of its combination of circular hide exterior and the four main struts of the interior wood supports. Waters (1963) provides an extensive illustration of the cultural significance of combining the circular and cross form in his commentary on the Hopi creation myth.

The fourfold symmetry of the quadrilateral form has lead to some sophisticated conceptual structures in Native American knowledge systems. In Navajo sand painting, for example, the cruciform shape represents the "four directions" concept, similar to the Cartesian coordinate system. While orderly and consistent, it is by no means simple (see Witherspoon and Peterson 1995). The four Navajo directions are also associated with corresponding sun positions (dawn, day, evening, night), yearly seasons (spring, summer, fall, winter), principal colors (white, blue, yellow, black), and other quadrilateral divisions (botanical categories, partitions of the life cycle, etc.). These are further broken into intersecting bipolarities (e.g., the east/west sun path is broken by the north/south directions). Combined with circular curves (usually representing organic shapes and processes), the resulting schema are rich cultural resources for indigenous mathematics (see Moore 1994). But, except for minor repetitions (like the small circular kivas in the Chaco canyon site of fig. 3.1) there is nothing particularly fractal about these quadrilateral designs.

Many Mesoamerican cities, such as the Mayans' Teotihuacán, the Aztec's Tenochtitlán, and the Toltec's Tula, embedded a wealth of astronomical knowledge in their rectangular layouts, aligning their streets and buildings with heavenly objects and events (Aveni 1980). J. Thompson (1970) and Klein (1982)

describe the quadrilateral figure as an underlying theme in Mesoamerican geometric thinking, from small-scale material construction techniques such as weaving, to the heavenly cosmology of the four serpents. Rogelio Díaz, of the Mathematics Museum at the University of Querétaro, points out that the skin patterns of the diamondback rattlesnake were used by the Mayans to symbolize this concept (fig. 3.3a).

Comparing the Mayan snake pattern with an African weaving based on the cobra skin pattern (fig. 3.3b), we can see how geometric modeling of similar natural phenomena in these two cultures results in very different representations. The Native American example emphasizes the Euclidean symmetry *within one* size frame ("size frame" because the term "scale" is confusing in the context of snake skin). This Mayan pattern is composed of four shapes of the same size, a fourfold symmetry. But the African example emphasizes fractal symmetry, which is not about similarity between right/left or up/down, but rather similarity *between different* size frames. The African snake pattern shows diamonds within diamonds within diamonds. Neither design is necessarily more accurate: cobra skin does indeed exhibit a fractal pattern—the snake's "hood," its twin white patches, and the individual scales themselves are all diamond shaped—and yet snake skin patterns (thanks to the arrangement of the scales) are also characteristically in diagonal rows, so they are accurately modeled as Euclidean structures as well. Each culture chooses to emphasize the characteristics that best fit its design theme.

There are a few cases in which Native Americans have used scaling geometries in artistic designs. Several of these were not, however, part of the traditional repertoire.[2] Navajo blankets, for example, were originally quite linear; it was only on examining Persian rugs that Navajo weavers began to use more scaling styles of design (and even then the designs were much more Euclidean than the Persian originals; see Kent 1985). The Pueblo "storyteller" figures have some scaling properties, but they are of recent (1960s) origin. Pottery and calabash (carved gourd) artisans in Africa often create scaling by allowing the design adaptively to change proportion according to the three-dimensional form on which it is inscribed (see "adaptive scaling" in chapter 6), but this was quite rare in Native American pottery until the 1960s.

Finally, there are three Native American designs that are both indigenous and fractal. The best case is the abstract figurative art of the Haida, Kwakiutl, Tlingit, and others in the Pacific Northwest (Holm 1965). The figures, primarily carvings, have the kind of global, nonlinear self-similarity necessary to qualify as fractals and clearly exhibit recursive scaling of up to three or four iterations. They also make use of adaptive scaling, as illustrated by the shrinking series of

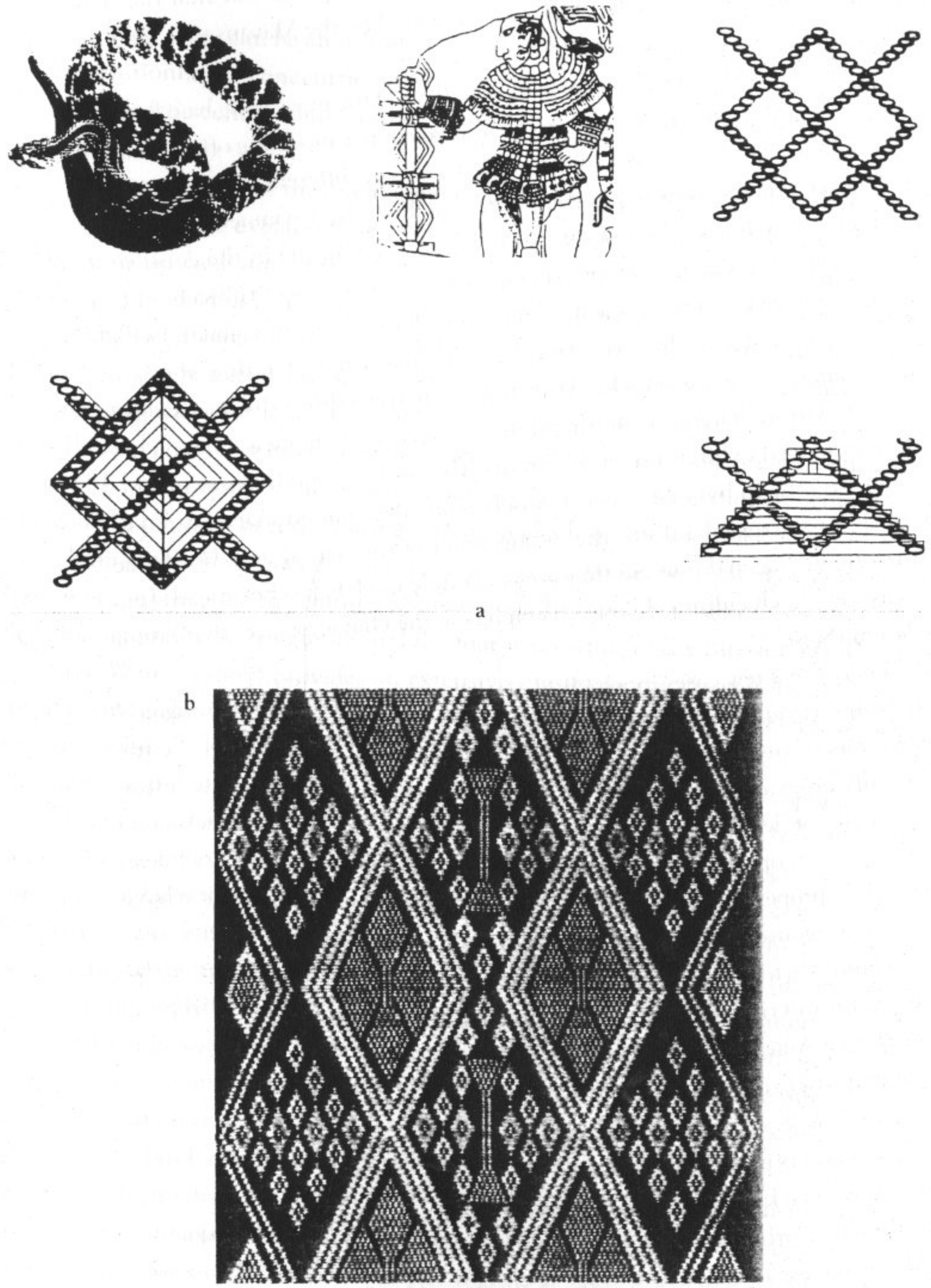


FIGURE 3.3

Snakeskin models in Native American and African cultures

(a) Rogelio Díaz of the Mathematics Museum at the University of Querétaro shows how the skin patterns of the diamondback rattlesnake were used by the Mayans to symbolize a cosmology based on quadrilateral structure. **(b)** The Mandiack weavers of Guinea-Bissau have also created an abstract design based on a snakeskin pattern, but chose to emphasize the fractal characteristics.

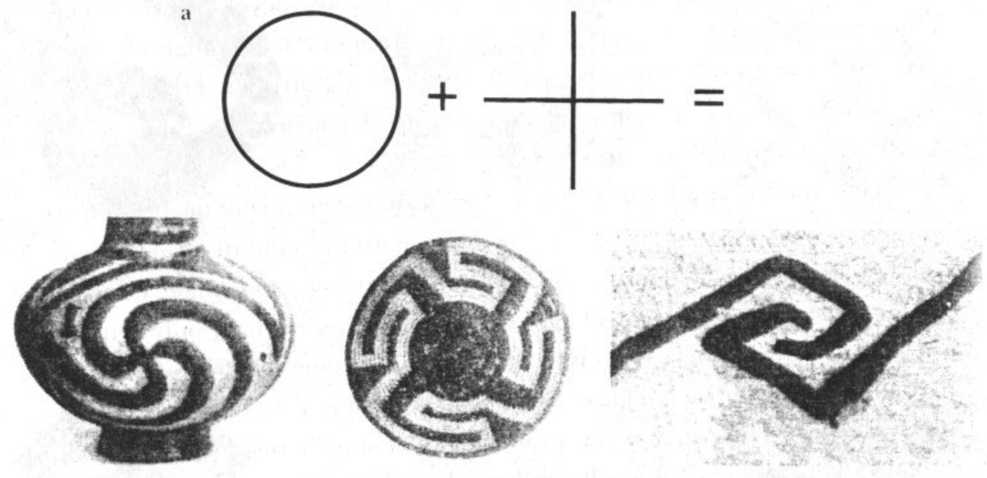
figures on the diminishing handles of soup ladles. Researchers since Adams (1936) have pointed to the similarity with early Chinese art, which also has some beautiful examples of scaling form, and its style of curvature and bilateral symmetry could indeed be culturally tied to these New World designs through an ancient common origin. However, I doubt that is the case for the scaling characteristics. The Pacific Northwest art appears to have developed its scaling structure as the result of competition between artisans for increasingly elaborate carvings (Faris 1983). Although some researchers have attributed the competition pressure to European trading influences, the development of the scaling designs was clearly an internal invention.

The other two traditional Native American designs do not qualify as fractals quite as well. One involves the saw-tooth pattern found in several basket and weaving designs. When two saw-tooth rows intersect at an angle, they create a triangle made from triangular edges. But these typically have only two iterations of scale, and there is no indication in the ethnographic literature that they are semantically tied to ideas of recursion or scaling (see Thomas and Slockish 1982, 18). The other is an arrangement of spiral arms often found on coiled baskets. Again, this is self-similar only with respect to the center point, but there are some nonlinear scaling versions (that is, designs that rapidly get smaller as you move from basket edge to center). However, these designs generally appear to be a fusion between the circular form of the basket and the cruciform shape of the arms: again more a combination of two Euclidean shapes than a fractal.

One of the most common examples of this fusion between the circle and the cross is the "bifold rotation" pattern in which the arms curve in opposite directions, as shown in figure 3.4a. Figure 3.4b shows the figure of a bat from Mimbres pottery with a more complex version of the bifold rotation. Euclidean symmetry has been emphasized in this figure; for example, the ears and mouth of the bat have been made to look similar to increase the bilateral symmetry, and the belly is drawn as a rectangle. Figure 3.4c shows the figure of a bat from an African design; it is a zigzag shape that expands in width from top to bottom, representing the wing of the bat. Here we see neglect of the bilateral symmetry of the bat, and an emphasis on the scaling folds of a single wing. Again, the Native American representation makes use of its quadrilateral/circular design theme, just as the African representation of the bat emphasizes scaling design.

There is plenty of complexity and sophistication in the indigenous geometry and numeric systems of the Americas (see Ascher 1991, 87–94; Closs 1986; Eglash 1998b), but with the impressive exception of the Pacific Northwest carvings, fractals are almost entirely absent in Native American designs.

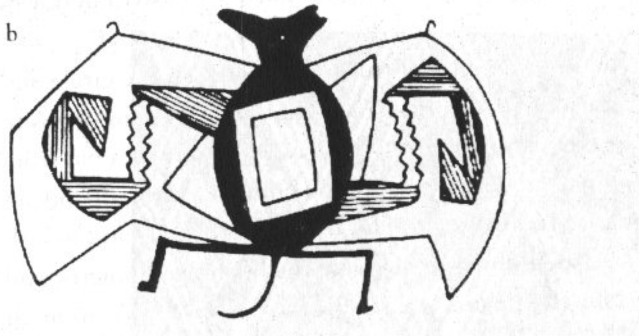
a



Arkansas pottery | Pima basket | Southwestern pottery motif

(a) The circular and quadrilateral forms were often combined in Native American designs as a fourfold or bifold rotation.

b



c

(b) This image of a bat, from a Mimbres pottery in Southwestern Native American tradition, shows an emphasis on circular and quadrilateral form. The ear and the mouth, for example, are made to look similar to emphasize bilateral symmetry, and the belly is drawn as a rectangle. It also shows the wing bones as a bifold rotation pattern.

(c) This African sculpture of a bat, from the Lega society of Zaire, pays little attention to the bilateral symmetry of the bat's body but gives an emphasis on the scaling symmetry of the wing folds, shown as an expanding zigzag pattern.

FIGURE 3.4

The bifold rotation in Native American design

(**a:** *Left, from Miles 1963. Center, from Southwest Indian Craft Arts by Clara Lee Tanner. Copyright 1968 by the Arizona Board of Regents. Reprinted by permission of the University of Arizona Press. Right, courtesy Don Crowe.* **b,** *from Zaslow 1977, courtesy of the author.* **c,** *courtesy of Daniel Biebuyck.*)

Designs of Asia and the South Pacific

Several of the South Pacific cultures share a tradition of decorative curved and spiral forms, which in certain Maori versions—particularly their rafter and tattoo patterns—would certainly count as fractal (see Hamilton 1977). These are strongly suggestive of the curvature of waves and swirling water. Classic Japanese paintings of water waves were also presented as fractal patterns in Mandelbrot's (1982) seminal text (plate C16). These may have some historic relation to scaling patterns in Chinese art (see Washburn and Crowe 1988, fig. 6.9), which are based on swirling forms of water and clouds, abstracted as spiral scaling structures. While both the Japanese and Chinese patterns are explicitly associated with an effort to imitate nature, these Maori designs are reported to be more about culture—in particular, they emphasize mirror-image symmetries, which are associated with their cultural themes of complimentarity in social relations (Hanson 1983).

In almost all other indigenous examples, however, the Pacific Islander patterns are quite Euclidean. Settlement layout, for instance, is typically in one or two rows of rectangular buildings near the coasts, with circular arrangements of rectangles also occurring inland (see Fraser 1968). The building construction is generally based on a combination of rectangular grids with triangular or curved arch roofs. Occasionally these triangular faces are decorated with triangles, but otherwise nonscaling designs dominate both structural and decorative patterns.[3]

Again, it is important to note that this lack of fractals does not imply a lack of sophistication in their mathematical thinking. For example, Ascher (1991) has analyzed some of the algorithmic properties of Warlpiri (Pacific Islander) sand drawings. Similar structures are also found in Africa, where they are called *lusona*. But while the lusona tend to use similar patterns at different scales (as we will see in chapter 5), the Warlpiri drawings tend to use different patterns at different scales. Ascher concludes that the Warlpiri method of combining different graph movements is analogous to algebraic combinations, but the African lusona are best described as fractals.

Complicating my characterization of the South Pacific as dominated by Euclidean patterns is the extensive influence of India. It is perhaps no coincidence that the triangle of triangles mentioned above is most common in Indonesia. In architecture, a famous exception to the generally Euclidean form is that of Borobudur, a temple of Indian religious origin in Java. Although northern India tends toward Euclidean architecture, explicit recursive design is seen in several temples in southern India—the Kandarya Mahadeo in Khajuraho is one of the clearest examples—and is related to recursive concepts in religious cosmology. These same areas in southern India also have a version of the lusona drawings, and many other examples of fractal design. Interestingly, these examples from southern India are the products of Dravidian culture, which is suspected to have significant historical roots in Africa.

European designs

Most traditional European fractal designs, like those of Japan and China, are due to imitation of nature—a topic we will take up in the following chapter. There are at least two stellar exceptions, however, that are worth noting. One is the scaling iterations of triangles in the floor tiles of the Church of Santa Maria in Cosmedin Rome (see plate 5.7 in Washburn and Crowe 1988). I have not been able to determine anything about their cultural origins, but they are clearly artistic invention rather than imitation of some natural form. The other can be found in certain varieties of Celtic interlace designs. Nordenfalk (1977) suggests that these are historically related to the spiral designs of pre-Christian Celtic religion, where they trace the flow of a vital life force. They are geometrically classified as an Eulerian path, which is closely associated with mathematical knot theory (cf. Jones 1990, 99).

Conclusion

Fractal structure is by no means universal in the material patterns of indigenous societies. In Native American designs, only the Pacific Northwest patterns show a strong fractal characteristic; Euclidean shapes otherwise dominate the art and architecture. Except for the Maori spiral designs, fractal geometry does not appear to be an important aspect of indigenous South Pacific patterns either. That is not to say that fractal designs appear nowhere but Africa—southern India is full of fractals, and Chinese fluid swirl designs and Celtic knot patterns are almost certainly of independent origin.[4] The important point here is that the fractal designs of Africa should not be mistaken for a universal or pancultural phenomenon; they are culturally specific. The next chapter will examine the question of their mathematical specificity.